Indepenent and Mutually Exclusive Events

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Independence and Mutual Exclusivity

In the study of statistics, these concepts always seem to arise at roughly the same time, but they are quite different. They are both ideas you are already familiar with, but you need to learn new language for expressing them.

Independence

Events are independent if they do not mathematically interact in any way. They have no effect on each other.

Mutual exclusivity

Events are disjoint or mutually exclusive if they can never happen together.

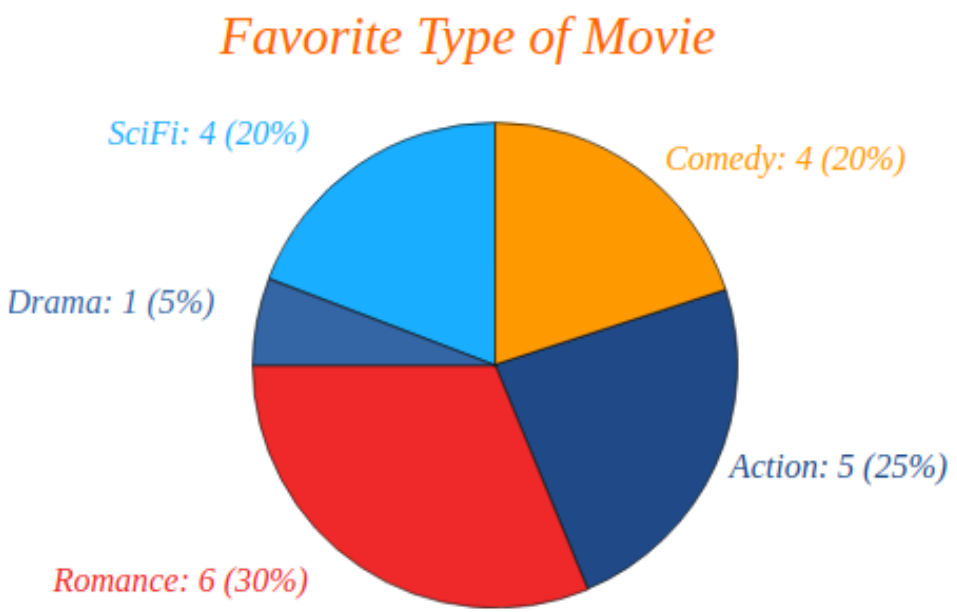
Example of Inpedendence

I got this pie chart from the **Math Is Fun** website:

Imagine you survey your friends to find the kind of movie they like best:

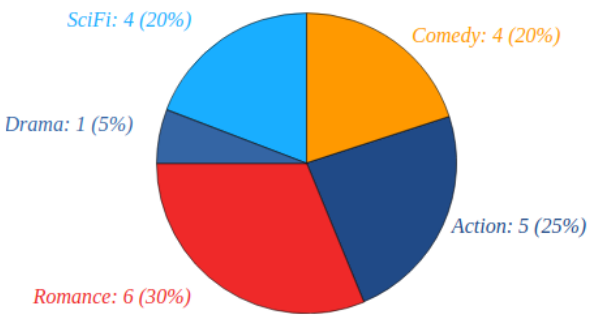
Table: Favorite Type of Movie				
Comedy	**Action**	**Romance**	**Drama**	**SciFi**
4	5	6	1	4

You can show the data by this Pie Chart:

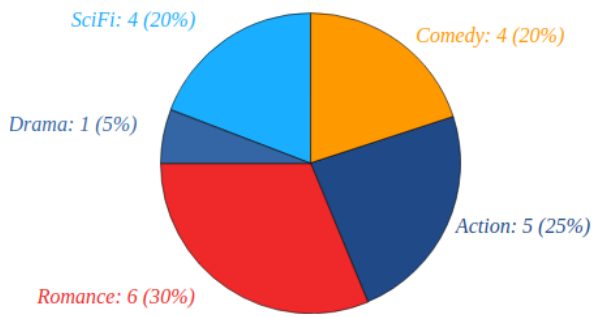


Example:

Let's pretend that instead of just doing this survey with 20 people, you did a survey of 20 men and 20 women, and the results ended up exactly the same for both groups. (I would suspect you faked your data, but that's a separate issue.)



MEN



WOMEN

Here, gender had no effect on movie preferences.
Movie preferences were independent from gender.

Testing for Independence

	A	B	C	D	E
P					
Q					
R					
S					
T					

Two events which can be represented by one row and one column in a two-way table may possibly be independent. If the actual counts are equal to the expected counts, in other words:

$$Count = \frac{RowSum \times ColumnSum}{OverallTotal},$$

then those events are independent and vice versa. If this relationship holds for the entire two-way table, then all the row events are independent from all the column events and vice versa.

Question:

Are color and truth value independent in this grid?

	True	False
Red	25	30
Blue	75	90

Answer:

Color and truth value ARE independent in this grid!

	True	False
Red	25	30
Blue	75	90

Balancing a 2x2 grid:

For a 2x2 grid, it is fine to show just one cell satisfies the formula. If one does, they all must. However, this only works for a 2x2 grid. Why does it work? Imagine you wish to subtract one from the 25. What would you need to do to balance out all the total counts (which must remain the same)?

	True	False
Red	25-1	30+1
Blue	75+1	90-1

For a 2x2 grid, if one cell is off balance, all other cells must be off balance by the same amount, with opposite corners in the same direction.

Many ways to show independence (proportionality):

- Notice that 25 x **3** = 75 and 30 x **3** = 90
- Notice that 55 x 100 / 220 = 25
- Notice that 25 x **1.2** = 30 and 75 x **1.2** = 90
- Notice that 25% of both True and False are Red
- Notice that 5/11 of both Red and Blue are True

	True	False
Red	25	30
Blue	75	90

The implication goes both ways

Formula implies independence

$$P(A) \times P(B) = P(A \text{ and } B)$$

When true, this implies events A and B are independent.

Independence implies formula

When events A and B are independent, then

$$P(A) \times P(B) = P(A \text{ and } B)$$

Disjoint Events

Disjoint events never happen together.

It cannot be summer and winter at the same time.
Those events are disjoint.

In case you were wondering

Independent events cannot also be disjoint, unless neither event can ever occur. (And that would be pointless!)

They occupy parallel columns (or rows)

	A	B	C	D	E
P					
Q					
R					
S					
T					

	A	B	C	D	E
P					
Q					
R					
S					
T					

Unlike with independence, you don't have to calculate anything! YAY! Just the fact that you can write these things in parallel columns (or rows) is enough!

It goes both ways again!

Formula implies DISJOINT

$$P(A) + P(B) = P(A \text{ or } B)$$

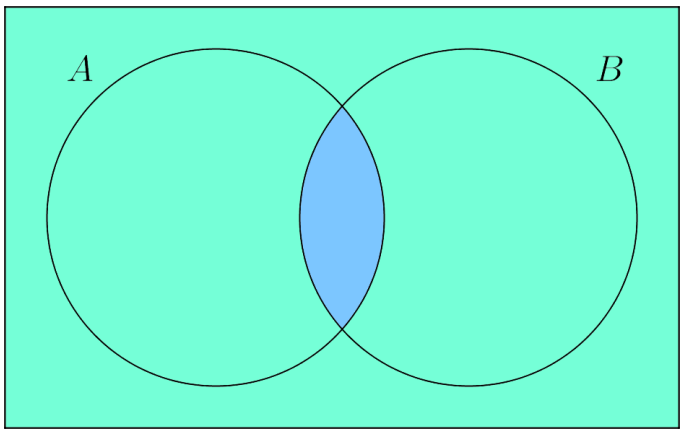
When true, this implies events A and B are disjoint.

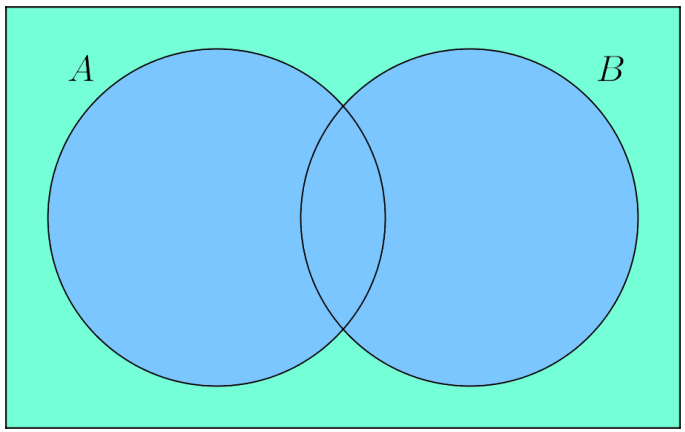
DISJOINT implies formula

When events A and B are disjoint, then

$$P(A) + P(B) = P(A \text{ or } B)$$

Generic Formulas





$$P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Disjoint case

The disjoint case is just a special case where there is no overlap or *no football-shaped region*.

Since there is no overlap, there is no probability of an **and** situation.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

becomes

$$P(A \text{ or } B) = P(A) + P(B)$$

Putting it into practice

Exercise: You have a box with 25 balls in it, identical except for color.
You have 10 red and 15 blue balls in the box.
Without looking, you select a ball, note its color, then return it to the box.

If you draw 3 balls in this manner, what's the probability they are all red?

Answer:

Since the draws are independent, you can multiply their probabilities together.

$$\frac{10}{25} \times \frac{10}{25} \times \frac{10}{25} = (.4)^3 = .064 = 6.4\%$$

Question:

Again with 10 red and 15 blue balls in a box:

We draw three balls as before. What is the probability of drawing 3 red or 3 blue balls?

Answer:

The probability of drawing three blue balls is

$$\frac{15}{25} \times \frac{15}{25} \times \frac{15}{25} = (.6)^3 = .216 = 21.6\%$$

Since the probability of drawing three blue balls is disjoint from drawing three red balls:

$$0.216 + 0.064 = 0.280 = 28\%.$$

Question:

With the same setup again and three draws: What is the probability of drawing at least one red ball?

Answer:

The probability of drawing all blue was 21.6%. All other cases contain at least one red ball.

$$1 - 0.216 = 0.784 = 78.4\%$$

Dice Questions

For a roll of two standard six-sided dice, let event A be: rolling two dice both showing even values. Let event B be: rolling two dice with at least one showing a prime number.

Are these events disjoint?

Are these events independent?

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Start with this grid!

Dice Questions

For a roll of two standard six-sided dice, let event A be: rolling two dice both showing even values. Let event B be: rolling two dice with at least one showing a prime number.

Are these events disjoint? Are these events independent?

	1	2	3	4	5	6
1						
2		A		A		A
3						
4		A		A		A
5						
6		A		A		A

Fill in the table with some symbol to represent the first event.

Dice Questions

For a roll of two standard six-sided dice, let event A be: rolling two dice both showing even values. Let event B be: rolling two dice with at least one showing a prime number.

Are these events disjoint? Are these events independent?

	1	2	3	4	5	6
1		B	B		B	
2	B	AB	B	AB	B	AB
3	B	B	B	B	B	B
4		AB	B	A	B	A
5	B	B	B	B	B	B
6		AB	B	A	B	A

Fill in the table with some symbol to represent the second event.

Answer questions

Are these events disjoint? Are these events independent?

	1	2	3	4	5	6
1		B	B		B	
2	B	AB	B	AB	B	AB
3	B	B	B	B	B	B
4		AB	B	A	B	A
5	B	B	B	B	B	B
6		AB	B	A	B	A

These events are **not disjoint** because **they can occur together**.

We now test for independence:

$$\frac{9}{36} \times \frac{27}{36} \stackrel{?}{=} \frac{5}{36}$$

Test for independence continued

Note that we had 9 instances of A, 27 instances of B, and 5 together.

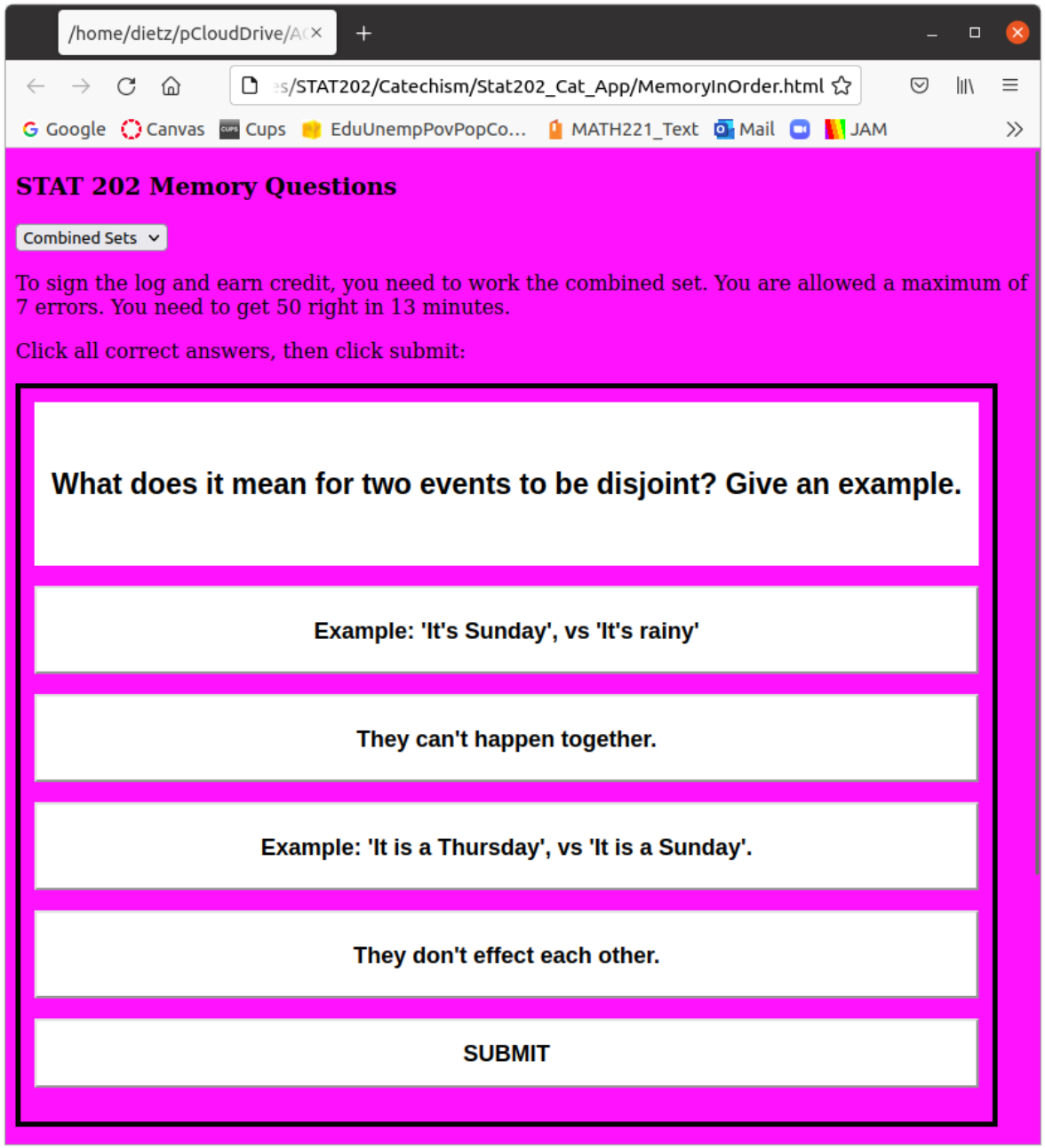
$$\frac{9}{36} \times \frac{27}{36} \stackrel{?}{=} \frac{5}{36}$$
$$\frac{1}{4} \times \frac{3}{4} \stackrel{?}{=} \frac{5}{36}$$
$$\frac{3}{16} \stackrel{?}{=} \frac{5}{36}$$

Since $\frac{3}{16}$ and $\frac{5}{36}$ are both in lowest terms, they can't be equal.

However, if you prefer to work with decimals, go ahead. You do not need to find lowest form for the fractions to show they are unequal if you instead convert to decimal.

MEMORY QUESTIONS

3 today!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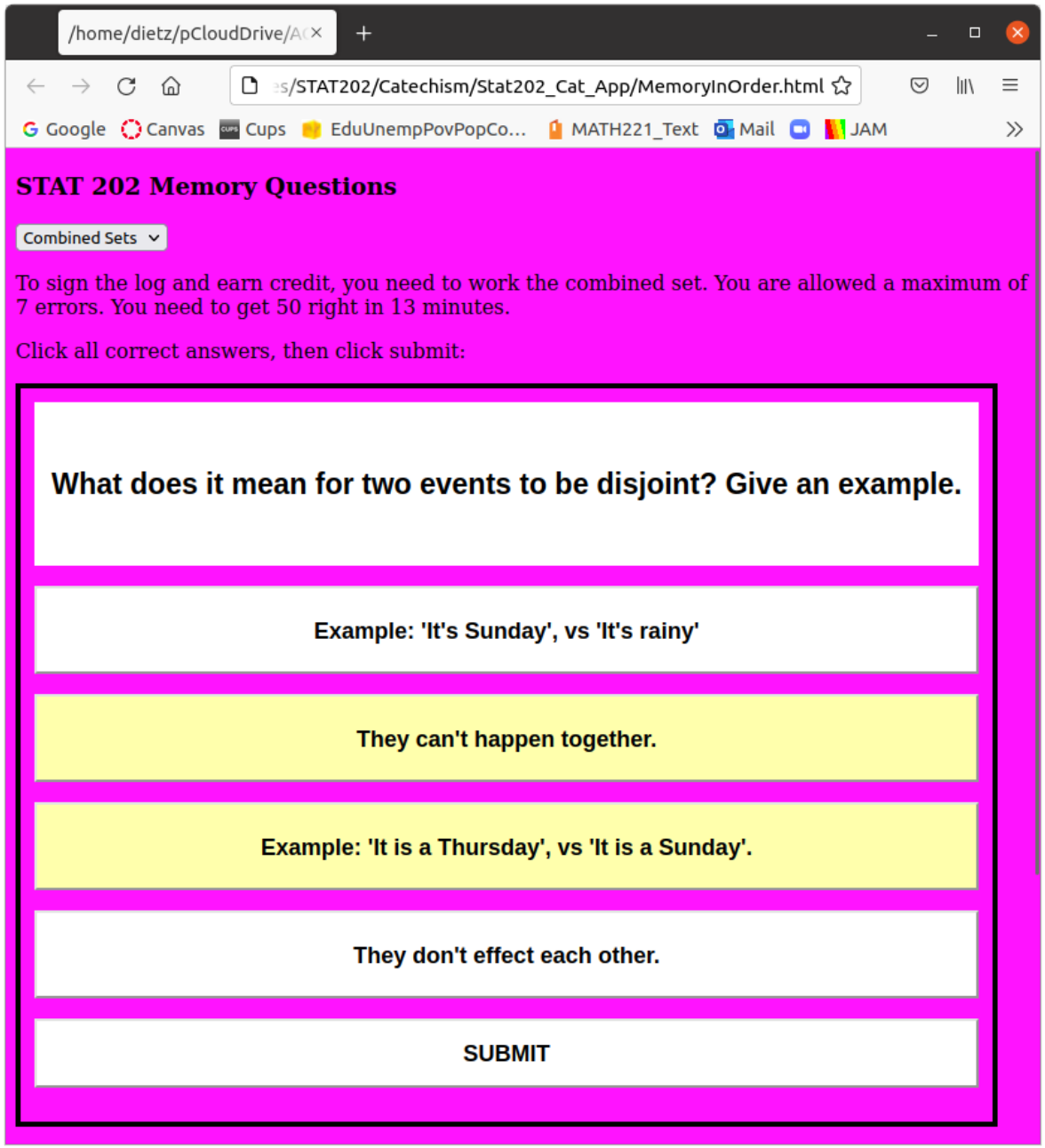
Combined Sets
To sign the log and earn credit, you need to work the combined set. You are allowed a maximum of 7 errors. You need to get 50 right in 13 minutes.
Click all correct answers, then click submit:
What does it mean for two events to be disjoint? Give an example.
Example: 'It's Sunday', vs 'It's rainy'
They can't happen together.
Example: 'It is a Thursday', vs 'It is a Sunday'.
They don't effect each other.
SUBMIT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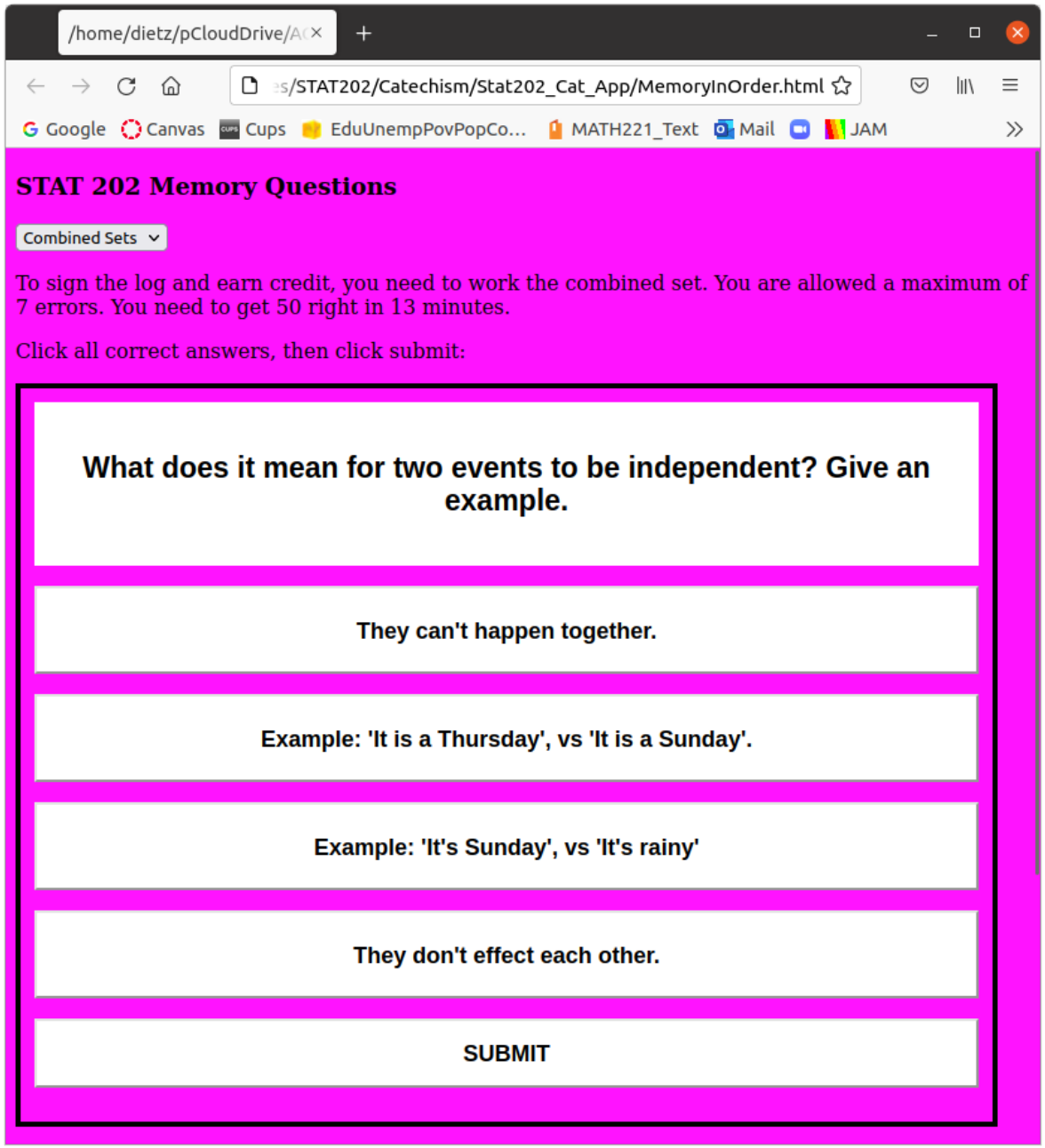
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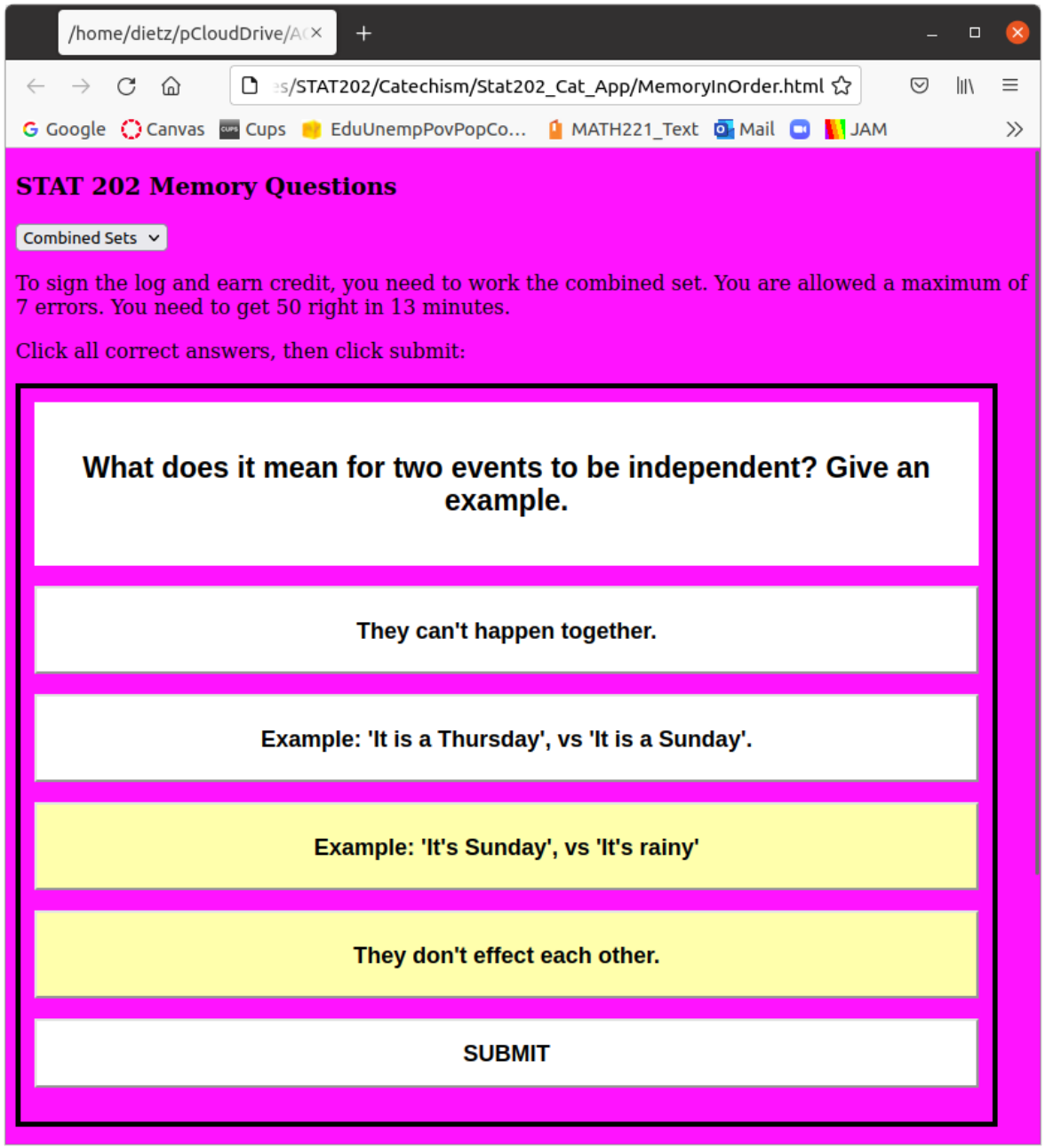
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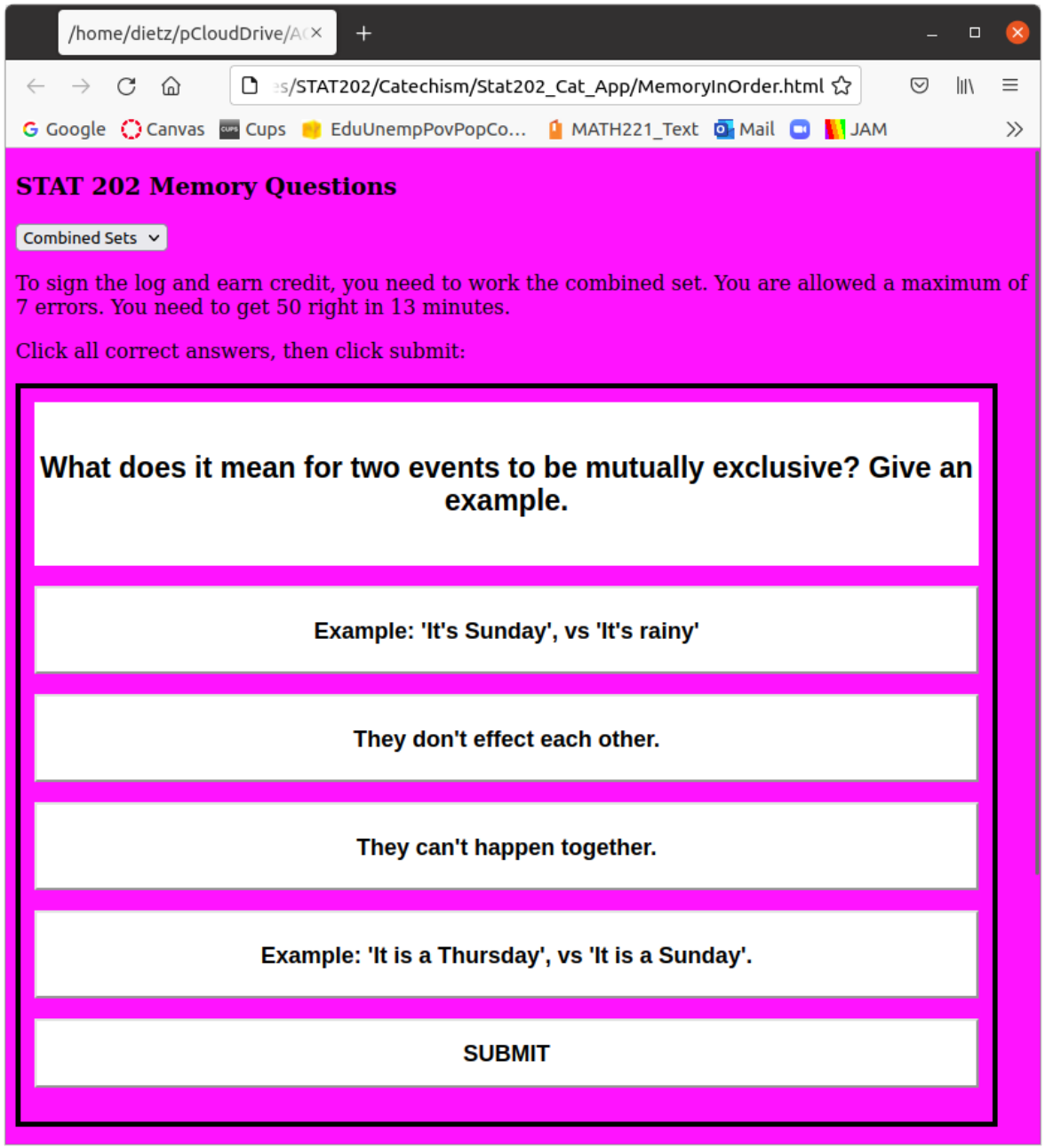
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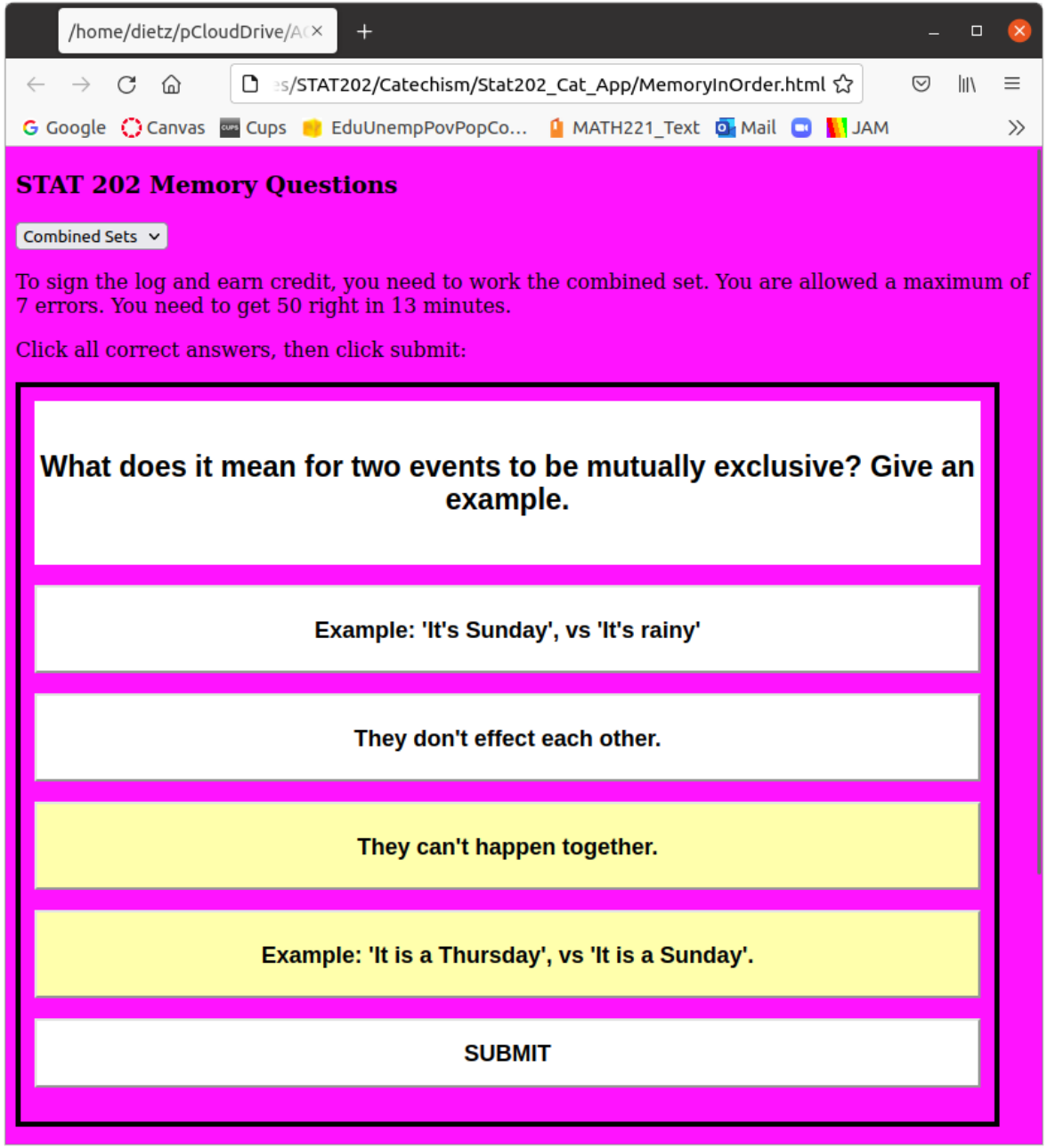
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