# Discrete Distributions 

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## Discrete Distributions

Here is a little food for thought:
Anything you can store in a computer is actually discrete!
A discrete distribution is a list or table of values a variable can have, together with its count in a population or its probability of occuring. Computers can only store zeros and ones. That's it! Of course, we turn those zeros and ones into more complex language structures so we can make references to things which are not discrete. But, in a very real way, there is a limit to what you can do on a computer.

## Examples

We often think of dice and spinners as representing discrete distributions.
Often we would like to work with something which is continuous by nature, but we collect data in a discrete way, and we compute in a discrete way. Sometimes we do this by choice, or for convenience. But sometimes there is no other option.

## Weather

At wunderground.com, you will see that the February 20 temperature at Ronald Reagan airport gives a daily average of 40F with a record high of 79F and a record low of 5 F .

For February 21, we have a daily average of 40F, record high of 82 F and record low of 7 F .

For February 22, we have an average of 41F, record high of 77 F , and record low of 9F.

The entire database the website sits on top of is made up of thousands upon thousands of records, all discrete, all trying to keep track of something which is by its nature a continuous phenomenon.

## Staring out

In this course, we use small discrete distributions to make pedagogical points.

## Finding one missing value

If you have a nearly complete distribution and only one count (or percent) is missing, but you know the total, you can recover that one value (or percent).

## Example

In a parking lot, you see 30 cars. How many have four seats?

| Seats | 2 | 3 | 4 | 5 and up |
| :--- | :--- | :--- | :--- | ---: |
| \# Cars | 4 | 0 | $?$ | 20 |

## Answer

In a parking lot, you see 30 cars. How many have four seats?

| Seats | 2 | 3 | 4 | 5 and up |
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The answer is 6, which you knew.

## Weighted averages

We can make use of discrete distributions to calculate weighted averages or expected values.

Let's pretend we did a quick poll of people in the class and asked how many pens and pencils they were carrying. What is the average number of pencils and pens carried by a student in the class?

| \# Pencils and Pens | 0 | 1 | 2 | 15 |
| :--- | :--- | ---: | ---: | ---: |
| Students | 3 | 10 | 5 | 1 |

$$
\frac{(0 \times 3)+(1 \times 10)+(2 \times 5)+(15 \times 1)}{(3+10+5+1)}=\frac{35}{19} \approx 2.33
$$

## Weighted averages

When you take a weighted average, you are still just adding up everything you have (in this case pencils and pens) and dividing by the number of values (students). However, your data are presented to you in a different format, so you have to get used to a slightly different process mechanically.

When you are using percents instead of values, the same idea applies.

## Spinners

Find the average value of the following spinner:


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## A special case

When your data are symmetric, your mean and median coincide, so you don't have to calculate the average!


Find the average (or expected) value of this distribution!

## Theoretical problems

In theoretical problems, we can just list all the outcomes which are already known to have equal probabilities. We can also combine outcomes and make tables, with the problems being very similar to what we did recently when we studied basic probability.

## Example

Flip a four sided die and a six-sided die. Make a histogram with the probabilities for each sum.

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Flip a four sided die and a six-sided die. Make a histogram with the probabilities for each sum.

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 2 | 3 | 4 | 5 | 6 | 7 |
| $\mathbf{2}$ | 3 | 4 | 5 | 6 | 7 | 8 |
| $\mathbf{3}$ | 4 | 5 | 6 | 7 | 8 | 9 |
| $\mathbf{4}$ | 5 | 6 | 7 | 8 | 9 | 10 |

## Table of outcomes

|  | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| chance out of $\mathbf{2 4}$ | 1 | 2 | 3 | 4 | 4 | 4 | 3 | 2 | 1 |

We again have symmetric data, so we can conclude that the average roll for this situation is a six. However, we can do it out the long way to demonstrate this more general process.

## The long method

$$
\begin{gathered}
\frac{2 \cdot 1+3 \cdot 2+4 \cdot 3+5 \cdot 4+6 \cdot 4+7 \cdot 4+8 \cdot 3+9 \cdot 2+10 \cdot 1}{24} \\
=6
\end{gathered}
$$

## MEMORY QUESTION



## STAT 202 Memory Questions

```
Combined Sets 
To sign the log and earn credit, you need to work the combined set. You are allowed a maximum of 7 errors. You need to get 50 right in 13 minutes.
Click all correct answers, then click submit:
```

If you roll two standard 6-sided dice, what is the expected value for the sum?

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## SUBMIT



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