Hypothesis tests for proportions using Confidence Intervals

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STAT 202 - Spring 2020

- This is an important day, as we introduce the long-awaited hypothesis testing! It will stay with us for the rest of the semester, and on into any other statistics course you may take!
- What is a hypothesis and why would we want to test it?

Dictionary

Search for a word Q hy·poth·e·sis /hī'päTHəsəs/ noun a supposition or proposed explanation made on the basis of limited evidence as a starting point for further investigation. "professional astronomers attacked him for popularizing an unconfirmed hypothesis" theorem thesis conjecture Similar: theory supposition speculation \sim PHILOSOPHY a proposition made as a basis for reasoning, without any assumption of its truth.

Translations, word origin, and more definitions

From Oxford

Feedback

A hypothesis is a starting point. It's a question phrased as an answer. If there is a reason or data behind it, it's often given more weight.

Special situations

Some special situations are easier to visualize.



I grab one of these dice and don't tell you which one it is. Roll after roll, numbers are called out, until you can eventually tell me which one I have! Perhaps you wonder if you made a specific mistake. There were two sets of keys hanging on the hook. Perhaps you took the wrong set?

In those special situations, there are only a few options. Once you eliminate the other/s from being likely, you can conclude that the remaining option is the only possibility.

Confidence intervals for proportions

$$p \mp z^* \sqrt{\frac{p(1-p)}{n}}$$

If you are estimating where your sample will land, you use p from your hypothesized value. If you already *have* a sample, swap p with \hat{p} , your sample proportion.

In this course, our usual situation is that we are making a guess at a population parameter and we want our sample statistics to tell us whether our guess was close or not.

Notice I said *close or not*.

Unless you have just a limited number of possibilities, you can never be sure your guess is **perfect**. You can only tell if you are **close**.

Here is an example:

- Andrew thinks 80% of the students like chocolate.
- Betty thinks 82% of the students like chocolate.
- Chris thinks 75% of the students like chocolate.
- Doris thinks 50% of the students like chocolate.

We take a survey of 50 students. 78% of our survey respondents say they like chocolate.

- (claim: 80%) Andrew's confidence interval: 68.7% to 91.3%
- (claim: 82%) Betty's confidence interval: 71.1% to 92.9%
- (claim: 75%) Chris' confidence interval: 62.8% to 87.2%
- (claim: 50%) Doris' confidence interval: 35.9% to 64.1%

The survey itself gave us 78%, yielding a different kind of confidence interval of 66.3% to 89.7%. This captured Andrew's, Betty's, and Chris' claims!

Also, Andrew, Betty's and Chris' confidence intervals captured the sample proportion!

Usually, but not always, the capture goes in both directions or fails in both directions. If it works in one direction and fails in the other, it's because you're really too close to tell.

When you have a hypothesis, you build a confidence interval for where you predict your sample statistic will end up.

When you have a sample, you build a confidence interval for where you think the population parameter might be!

- Andrew could be right.
- Betty could be right.
- Chris could be right.
- Doris is nearly certainly wrong.

We can reject the Doris Hypothesis with 95% confidence! The others are still in the running, like candidates in a primary race! It's still possible that Andrew, Betty, and Chris are all off, and there was just a weird sample. This is why we hedge our bets by saying "with 95% confidence".

With our data in hand, we are fairly sure about Doris. But we can still never be sure about the others.

Unlike in the dice question, even if we were to eliminate two other options, there is no reason to believe Betty (or whomever) had any inside knowledge that would make them somehow *know* the true proportion.

When someone makes a claim that we might be trying to test (challenge, refute), then we usually give their claim the name "Null Hypothesis". This means that we are prepared to walk away from the argument if we can't prove it is wrong. We can never defend it or prove it's true, but failure to disprove it will give it more weight.

We say "H naught" or "null hypothesis" or "the null".

The little zero means "nothing", indicating the default assumptions or the status quo. Some students will say "H zero", but this terminology is discouraged.

Type I error

If H_0 is rejected when it's true, this is a Type I error. You accuse a friend of joking/lying when the friend is telling the truth.

Type II error

If there is insufficient evidence to reject H_0 , but it's actually false, a Type II error occurs. You accept (or pretend to accept) a lie.

Sadly, you can never tell how likely an error is!

We can make this statement: For a 99% confidence level, when the null hypothesis is true, we reject it about 1% of the time.



Let's say that H_0 is true, but we've been tricked, because we think we are far into a tail. This shows you when you're going to make a Type I error.



Let's say that H_0 is false, and some H_A is true, and in this diagram, it is above our cutoff value in our interval. We've been tricked, because we think we are nowhere near a tail. This shows you what a Type II error looks like if the actual mean is above the hypothesized one.



But you can't really tell what these diagrams are going to look like together, because you have no idea whether H_0 is true, and if it's false, you have even less of an idea for where the H_A curve goes, because it can be literally anyhwere!

No matter how hard we try, we can't know what we don't know.

We can take heart in the fact that it's at least a so-called known unknown.

Donald Rumsfeld 2002

As we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns, the ones we don't know we don't know.

Many found amusement from the manner in which Rumsfeld expressed this obvious fact.

In most cases, we don't have a finite number of alternatives. If we can't lay out the alternatives, we can't create a two-way table or determine the probability of making a Type II error. We can only give the probability of a Type I error, with the assumption that H_0 is true.

The very thing we are looking for in the first place is also what we need to know in order to make an accurate guess about the probability of a Type II error!

from intuitor.com



We can hope, at least, that we can bring in evidence that will separate the alternatives as far from each other as possible to reduce both types of errors.



Impact of Testimony

But often, what we end up with unwittingly is something like this, where H_A is surprisingly close, just because we have no clue where it is!

Let's play a game. We will secretly know what p is but we will hypothesize that it's really p = 0.67. Now that we know what p really is and also what we *think* it is, we can figure out precisely what's likely to happen when we draw a sample, say n = 50 and do a hypothesis test, let's say at 95%.

A common diagram looks something like this one:

10	COMES	The Null Hypothesis	The Alternative
		Is True	Hypothesis is True
	The Null Hypothesis Is True	Accurate	Type II Error
		1-α	β
		$\left(\cdot \cdot \right)$	(\cdot, \cdot)
		\bigcirc	\bigcirc
		Type I Error	Accurate
	The Alternative	α	1-β
	Hypothesis is True	$\left(\cdot \cdot \right)$	(\cdot, \cdot)
		\bigtriangledown	



Example: p = 0.67

If we have n = 50, we can think of this as a binomial problem (which it actually is) or we can do as the statisticians do and approximate it with the normal where $\mu = 0.67$ and $\sigma = \sqrt{0.67 \cdot 0.33/50}$, assume a 95% confidence level, and determine that we are in fact roughly 95% likely to make the correct decision and about 5% likely to make the wrong call and come up with a Type I error.





We can't proceed unless we know what the true proportion is.

Since H_0 is actually false, any claim that it is correct is suddenly wrong!



We have about a 90% chance of failure to reject it. But did we really want to reject it, because it's pretty close, isn't it?

Since H_0 is actually false, any claim that it is correct is wrong!



We have about a 95% chance of failure to reject it. This is even more insane than the previous example! We are super close! Did we really want to reject this?



The problem: We talk about these things without fully appreciating the implications of our (crazy) definitions!

I COMES	The Null Hypothesis	The Alternative Hypothesis is True
The Null Hypothesis Is True	Accurate 1 - α	Type II Error β
The Alternative Hypothesis is True	Type I Error α	Accurate $1 - \beta$

What we really need is an honest-to-goodness *margin of error*, but for whatever reason, when we do hypothesis testing, we don't bother. We let the confidence interval double as our margin of error.

As if there aren't enough issues already, a common thing to do in introductory statistics is to run one-tail tests. I am totally against this, and I view it (in nearly all cases) as an abuse of statistics. I'm not alone in this view.

It's totally reasonable to ask questions such as:

If we randomly sample from a population with a known distribution, what is the chance of our sample average winding up over a certain value?

Example: If you have a bowl of cookies whose weights are in the distribution N(100, 15), and you grab a sample of size 36, what's the chance that your average cookie weight will be under 95 grams?

A politician's polling numbers have been stable at around 60% for awhile. An agency is hired to run ads to increase the polling numbers.

Some people will run a one-tail test to see if the agency's efforts have resulted in an increase in the politician's popularity!

But it's a setup for a lie



By setting this up as a one-tail test, the statisticians are ignoring the very real possiblity that there is evidence that the ads had exactly the opposite of the desired effect! In fact, they've decided in advance that if this is the case, they will never reveal this result!

Another example: I give a pre-test to students before putting them through a study session. I then give them a post-test, and I decide to run the numbers as a one-tail test. I'm deciding in advance that there is no way possible that I've confused the students sufficiently that their scores have gone down.

If researchers are this confident that the opposite results are not possible, why aren't they confident enough to allow the research to demonstrate that for them!?



Well at least someone agrees with me! Who is she anyhow?

Karen was a statistical consultant at Cornell University for seven years before founding The Analysis Factor. Karen has worked with clients from undergraduate honor's students ... to tenured Ivy League professors, as well as non-profits and businesses...

Before consulting, Karen taught statistics courses for economics, psychology, and sociology majors at the University of California, Santa Barbara and Santa Barbara City College...

(And so on.)

Hypothesis testing using intervals:

- List the null and alternative hypotheses
- The null always contains equals
- The alternative negates the null
- Calculate the confidence interval
- If your sample statistic lies inside the interval
 - Fail to reject H_0
- If your sample statistic lies outside the interval
 - Reject H₀
 - Consider whether you care about the direction of the difference
- Rephrase your results in plain language for all to understand

Word Problem Template

Someone somewhere makes a statement about a population proportion. A researcher wants to test it and makes a study using a specified sample size of the population and gets a certain sample proportion. State the hypotheses, do the calculations, make a decision, and rephrase it.



The manager of the Piggly Wiggly states that 90% of his customers like to bake cookies at home. One of his assistant managers expresses doubt that this is correct. She takes a sample of 100 random shoppers and asks them. Of them, 75% admit to baking cookies at home. Did she disprove the manager? Use a 95% confidence level.

 $H_0: p = 0.9$ $H_A: p \neq 0.9$

Confidence interval for hypothesized value:

$$0.9 \mp 2\sqrt{0.9 \cdot 0.1/100} pprox 84\%$$
 to 96%

Our sample gave us $\hat{p} = 0.75$ or 75% which is not inside our interval, so we may not only reject the Null Hypothesis, but we may make a claim that the manager overestimated the proportion of customers who bake cookies at home.

Sometimes, you will want to make a very conservative estimate about your confidence interval for your proportion, or you may wish to determine a sample size in anticipation of a study whose purpose is to determine that! When this happens, **use** p = 0.5 **as a conservative estimate**. This will give you the widest possible interval, a slightly larger standard deviation, or a larger (more conservative) sample size.

The power of a test

The rate of Type II error is called "beta" or β , and we hope it's small like α is small. The **power** of a statistical test is $1 - \beta$ which is the power to correctly reject H_0 when it's false.

Question: What easy thing could you do to make the power of a test 100% and why would it be a bad idea?

XKCD Green Jelly Beans



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PERMANENT LINK TO THIS COMIC: HTTPS://XKCD.COM/882/ IMAGE URL (FOR HOTLINKING/EMBEDDING): HTTPS://IMGS.XKCD.COM/COMICS/SIGNIFICANT.PNG It's said that if the joke needs to be explained.... Well, go to https://www.explainxkcd.com/wiki/index.php/882:_Significant

8 MEMORY QUESTIONS































