# Hypothesis tests for proportions using Confidence Intervals 

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STAT 202 - Spring 2020

## What is a hypothesis

This is an important day, as we introduce the long-awaited hypothesis testing! It will stay with us for the rest of the semester, and on into any other statistics course you may take!

What is a hypothesis and why would we want to test it?

## A hypothesis is a guess

## Dictionary

## Search for a word <br> (4) hy-poth $e$.sis <br> /hī'päTHasas/ <br> noun

a supposition or proposed explanation made on the basis of limited evidence as a starting point for further investigation.
"professional astronomers attacked him for popularizing an unconfirmed hypothesis"
Similar: theory theorem thesis conjecture supposition speculation

- PHILOSOPHY
a proposition made as a basis for reasoning, without any assumption of its truth.

Translations, word origin, and more definitions

## Your guess is as good as mine!

A hypothesis is a starting point. It's a question phrased as an answer.
If there is a reason or data behind it, it's often given more weight.

## Special situations

Some special situations are easier to visualize.


I grab one of these dice and don't tell you which one it is. Roll after roll, numbers are called out, until you can eventually tell me which one I have!

## Special situations

Perhaps you wonder if you made a specific mistake. There were two sets of keys hanging on the hook. Perhaps you took the wrong set?

In those special situations, there are only a few options. Once you eliminate the other/s from being likely, you can conclude that the remaining option is the only possibility.

## Our formula for finding confidence intervals

## Confidence intervals for proportions

$$
p \mp z^{*} \sqrt{\frac{p(1-p)}{n}}
$$

If you are estimating where your sample will land, you use $p$ from your hypothesized value. If you already have a sample, swap $p$ with $\hat{p}$, your sample proportion.

## Our usual situation

In this course, our usual situation is that we are making a guess at a population parameter and we want our sample statistics to tell us whether our guess was close or not.

Notice I said close or not.
Unless you have just a limited number of possibilities, you can never be sure your guess is perfect. You can only tell if you are close.

## Close or not, not exact

Here is an example:

- Andrew thinks $80 \%$ of the students like chocolate.
- Betty thinks $82 \%$ of the students like chocolate.
- Chris thinks 75\% of the students like chocolate.
- Doris thinks $50 \%$ of the students like chocolate.

We take a survey of 50 students. $78 \%$ of our survey respondents say they like chocolate.

## Close or not, not exact

- (claim: 80\%) Andrew's confidence interval: $68.7 \%$ to $91.3 \%$
- (claim: $82 \%$ ) Betty's confidence interval: $71.1 \%$ to $92.9 \%$
- (claim: 75\%) Chris' confidence interval: $62.8 \%$ to $87.2 \%$
- (claim: $50 \%$ ) Doris' confidence interval: $35.9 \%$ to $64.1 \%$

The survey itself gave us $78 \%$, yielding a different kind of confidence interval of $66.3 \%$ to $89.7 \%$. This captured Andrew's, Betty's, and Chris' claims!

Also, Andrew, Betty's and Chris' confidence intervals captured the sample proportion!

## Caution:

Usually, but not always, the capture goes in both directions or fails in both directions. If it works in one direction and fails in the other, it's because you're really too close to tell.

When you have a hypothesis, you build a confidence interval for where you predict your sample statistic will end up.

When you have a sample, you build a confidence interval for where you think the population parameter might be!

## Results

- Andrew could be right.
- Betty could be right.
- Chris could be right.
- Doris is nearly certainly wrong.

We can reject the Doris Hypothesis with $95 \%$ confidence!
The others are still in the running, like candidates in a primary race!

## What if Doris is right though?

It's still possible that Andrew, Betty, and Chris are all off, and there was just a weird sample. This is why we hedge our bets by saying "with $95 \%$ confidence".

With our data in hand, we are fairly sure about Doris. But we can still never be sure about the others.

Unlike in the dice question, even if we were to eliminate two other options, there is no reason to believe Betty (or whomever) had any inside knowledge that would make them somehow know the true proportion.

When someone makes a claim that we might be trying to test (challenge, refute), then we usually give their claim the name "Null Hypothesis". This means that we are prepared to walk away from the argument if we can't prove it is wrong. We can never defend it or prove it's true, but failure to disprove it will give it more weight.

We say "H naught" or "null hypothesis" or "the null".
The little zero means "nothing", indicating the default assumptions or the status quo. Some students will say "H zero", but this terminology is discouraged.

## Type I and Type II Errors

## Type I error

If $H_{0}$ is rejected when it's true, this is a Type I error. You accuse a friend of joking/lying when the friend is telling the truth.

## Type II error

If there is insufficient evidence to reject $H_{0}$, but it's actually false, a Type II error occurs. You accept (or pretend to accept) a lie.

## How likely is it?

Sadly, you can never tell how likely an error is!
We can make this statement: For a $99 \%$ confidence level, when the null hypothesis is true, we reject it about $1 \%$ of the time.

## Type I error



## Type I error

Let's say that $H_{0}$ is true, but we've been tricked, because we think we are far into a tail. This shows you when you're going to make a Type I error.

## But Type II

## Type II error

Let's say that $H_{0}$ is false, and some $H_{A}$ is true, and in this diagram, it is above our cutoff value in our interval. We've been tricked, because we think we are nowhere near a tail. This shows you what a Type II error looks like if the actual mean is above the hypothesized one.

## Overlap



Type II error: Type I error

But you can't really tell what these diagrams are going to look like together, because you have no idea whether $H_{0}$ is true, and if it's false, you have even less of an idea for where the $H_{A}$ curve goes, because it can be literally anyhwere!

## We can't get around this

No matter how hard we try, we can't know what we don't know.
We can take heart in the fact that it's at least a so-called known unknown.

## Donald Rumsfeld 2002

As we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns, the ones we don't know we don't know.

Many found amusement from the manner in which Rumsfeld expressed this obvious fact.

## We don't know $H_{A}$

In most cases, we don't have a finite number of alternatives. If we can't lay out the alternatives, we can't create a two-way table or determine the probability of making a Type II error. We can only give the probability of a Type I error, with the assumption that $H_{0}$ is true.

The very thing we are looking for in the first place is also what we need to know in order to make an accurate guess about the probability of a Type II error!

## from intuitor.com



We can hope, at least, that we can bring in evidence that will separate the alternatives as far from each other as possible to reduce both types of errors.

## from intuitor.com



But often, what we end up with unwittingly is something like this, where $H_{A}$ is surprisingly close, just because we have no clue where it is!

## Let's pretend!

Let's play a game. We will secretly know what $p$ is but we will hypothesize that it's really $p=0.67$. Now that we know what $p$ really is and also what we think it is, we can figure out precisely what's likely to happen when we draw a sample, say $n=50$ and do a hypothesis test, let's say at $95 \%$.

## Data Science Dojo Blog

A common diagram looks something like this one:

| HYPOTHESIS TESTING OUTCOMES |  | Reality |  |
| :---: | :---: | :---: | :---: |
|  |  | The Null Hypothesis Is True | The Alternative Hypothesis is True |
| R |  | Accurate $1-\alpha$ | Type II Error $\beta$ |
| a |  | Type I Error | Accurate |
| $\begin{aligned} & \mathrm{c} \\ & \mathrm{~h} \end{aligned}$ | The Alternative <br> Hypothesis is True |  |  |

## So, if $H_{0}$ is true:



## Example: $p=0.67$

If we have $n=50$, we can think of this as a binomial problem (which it actually is) or we can do as the statisticians do and approximate it with the normal where $\mu=0.67$ and $\sigma=\sqrt{0.67 \cdot 0.33 / 50}$, assume a $95 \%$ confidence level, and determine that we are in fact roughly $95 \%$ likely to make the correct decision and about $5 \%$ likely to make the wrong call and come up with a Type I error.


$\mathrm{m}=\begin{gathered}50 \\ \mathrm{~F}(27 \quad \mathrm{p}: 0.67 \\ \text { Compute }\end{gathered}$

## So, if $H_{0}$ is false:



We can't proceed unless we know what the true proportion is.

## If $p=0.62$ but we suspect it's $p=0.67$.

Since $H_{0}$ is actually false, any claim that it is correct is suddenly wrong!


We have about a $90 \%$ chance of failure to reject it. But did we really want to reject it, because it's pretty close, isn't it?

## If $p=0.65$ but we suspect it's $p=0.67$.

Since $H_{0}$ is actually false, any claim that it is correct is wrong!



We have about a $95 \%$ chance of failure to reject it. This is even more insane than the previous example! We are super close! Did we really want to reject this?

## Problem: $H_{0}$ is never true!!!



The problem: We talk about these things without fully appreciating the implications of our (crazy) definitions!

## What we really need:



What we really need is an honest-to-goodness margin of error, but for whatever reason, when we do hypothesis testing, we don't bother. We let the confidence interval double as our margin of error.

## One tail versus two tail hypotheses

As if there aren't enough issues already, a common thing to do in introductory statistics is to run one-tail tests. I am totally against this, and I view it (in nearly all cases) as an abuse of statistics. I'm not alone in this view.

## This is fine:

It's totally reasonable to ask questions such as:
If we randomly sample from a population with a known distribution, what is the chance of our sample average winding up over a certain value?

Example: If you have a bowl of cookies whose weights are in the distribution $N(100,15)$, and you grab a sample of size 36 , what's the chance that your average cookie weight will be under 95 grams?

## But this is not:

A politician's polling numbers have been stable at around $60 \%$ for awhile. An agency is hired to run ads to increase the polling numbers.

Some people will run a one-tail test to see if the agency's efforts have resulted in an increase in the politician's popularity!

## But it's a setup for a lie



By setting this up as a one-tail test, the statisticians are ignoring the very real possiblity that there is evidence that the ads had exactly the opposite of the desired effect! In fact, they've decided in advance that if this is the case, they will never reveal this result!

## Another example

Another example: I give a pre-test to students before putting them through a study session. I then give them a post-test, and I decide to run the numbers as a one-tail test. I'm deciding in advance that there is no way possible that I've confused the students sufficiently that their scores have gone down.

If researchers are this confident that the opposite results are not possible, why aren't they confident enough to allow the research to demonstrate that for them!?

## Karen Grace-Martin



## One-tailed and Two-tailed Tests

by KAREN GRACE-MARTIN

I was recently asked about when to use one and two tailed tests.

The long answer is: Use one tailed tests when you have a specific hypothesis about the direction of your relationship. Some examples include you hypothesize that one group mean is larger than the other; you hypothesize that the correlation is positive; you hypothesize that the proportion is below .5.

The short answer is: Never use one tailed tests.

Why?

1. Only a few statistical tests even can have one tail: $z$ tests and $t$ tests. So you're severely limited. F tests, Chi-square tests, etc. can't accommodate one-tailed tests because their distributions are not symmetric. Most statistical methods, such as regression and ANOVA, are based on these tests, so you will rarely have the chance to implement them.
2. Probably because they are rare, reviewers balk at one-tailed tests. They tend to assume that you are trying to artificially boost the power of your test. Theoretically, however, there is

## Well at least someone agrees with me! Who is she anyhow?

## Karen Grace-Martin

Karen was a statistical consultant at Cornell University for seven years before founding The Analysis Factor. Karen has worked with clients from undergraduate honor's students ... to tenured Ivy League professors, as well as non-profits and businesses...

Before consulting, Karen taught statistics courses for economics, psychology, and sociology majors at the University of California, Santa Barbara and Santa Barbara City College...
(And so on.)

## Steps we will take:

Hypothesis testing using intervals:

- List the null and alternative hypotheses
- The null always contains equals
- The alternative negates the null
- Calculate the confidence interval
- If your sample statistic lies inside the interval
- Fail to reject $H_{0}$
- If your sample statistic lies outside the interval
- Reject $H_{0}$
- Consider whether you care about the direction of the difference
- Rephrase your results in plain language for all to understand


## Example:

## Word Problem Template

Someone somewhere makes a statement about a population proportion. A researcher wants to test it and makes a study using a specified sample size of the population and gets a certain sample proportion. State the hypotheses, do the calculations, make a decision, and rephrase it.

## Example

The manager of the Piggly Wiggly states that $90 \%$ of his customers like to bake cookies at home. One of his assistant managers expresses doubt that this is correct. She takes a sample of 100 random shoppers and asks them. Of them, $75 \%$ admit to baking cookies at home. Did she disprove the manager? Use a $95 \%$ confidence level.

## Example

$H_{0}: p=0.9$
$H_{A}: p \neq 0.9$
Confidence interval for hypothesized value:

$$
0.9 \mp 2 \sqrt{0.9 \cdot 0.1 / 100} \approx 84 \% \text { to } 96 \%
$$

Our sample gave us $\hat{p}=0.75$ or $75 \%$ which is not inside our interval, so we may not only reject the Null Hypothesis, but we may make a claim that the manager overestimated the proportion of customers who bake cookies at home.

## When no $p$ guess is available

Sometimes, you will want to make a very conservative estimate about your confidence interval for your proportion, or you may wish to determine a sample size in anticipation of a study whose purpose is to determine that! When this happens, use $p=0.5$ as a conservative estimate. This will give you the widest possible interval, a slightly larger standard deviation, or a larger (more conservative) sample size.

## Some more terminology

## The power of a test

The rate of Type II error is called "beta" or $\beta$, and we hope it's small like $\alpha$ is small. The power of a statistical test is $1-\beta$ which is the power to correctly reject $H_{0}$ when it's false.

Question: What easy thing could you do to make the power of a test $100 \%$ and why would it be a bad idea?

## XKCD Green Jelly Beans

## Significant



WE FOUND NO LINK BETWEEN PINK JELUY BEANS PND AONE ( $P>0.05$ )


WE FOUNDNO LINK BETWEEN TEAL JELIY BEANS PND AONE ( $P>0.05$ ).


## XKCD Green Jelly Beans



WE FOUNDNO LINK BETWEEN YELOW JELY BEANS AND AONE ( $P>0.05$ ).


## XKCD Green Jelly Beans



PERMANENT LINK TO THIS COMIC: HTTPS://XKCD.COM/882/
IMAGE URL (FOR HOTLINKING/EMBEDDING): HTTPS://IMGS.XKCD.COM/COMICS/SIGNIFICANT.PNG

## explaining....

It's said that if the joke needs to be explained.... Well, go to https://www.explainxkcd.com/wiki/index.php/882:_Significant

## 8 MEMORY QUESTIONS





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Combined Sets $\vee$

What is the alternative hypothesis?

It's the claim that something is causing the claim to differ from the observation, in other words, that the claim is wrong.

This says that the null hypothesis is incorrect.

It's the same as the null hypothesis.

It's the claim that is being made about how reality is set up.

SUBMIT

STAT 202 Memory Questions
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SUBMIT

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STAT 202 Memory Questions
Combined Sets ~
Can you use more than one null hypothesis at a time using basic STAT 202 tools?
```

Sure, but only two.

No, you can only have one null hypothesis.

Sure, but no more than 4.

You can have as many null hypotheses as you wish.

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To sign the log and eam oredit, you need to work the combined set. You are allowed a maximum of
7 orrors. You need to get 50 right in 13 minutes.
Click all correct answers, then click submit:

Can ever you have more than one hypothesis for a given set of data?

We can discuss the claims one by one.

We don't talk about multiple hypotheses in introductory courses.

No. You would need a new study with new data for each claim.

Yes, but it's often awkward to talk about them this way.

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You can demonstrate that will to persevere matters in statistics.

There are ways to prevent this from becoming an issue, but we don't discuss them at this level of study.

You may seemingly demonstrate an alternative hypothesis to be true, just because you've keep trying so much!

There is no problem doing this, just go right ahead.

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STAT 202 Memory Questions
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When doing hypothesis testing, what type of statement is assigned to the null hypothesis versus what is assigned to the alternative hypothesis?
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The alternative hypothesis supports the claim.

The null hypothesis is usually the 'boring' option.

The null hypothesis is the option that counters the claim.

The alternative is that something odd is happening.

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In hypothesis testing, who has the burden of proof?

We behave as if the null hypothesis is true unless proven false.

The person making the counter-claim has the burden of proof.

We presume the null hypothesis is false until proven true.

We presume the alternative hypothesis is true until proven false.

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