# When variances don't add 

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## Review Question: LSR does what?

Least Squares Regression (LSR, or a line of best fit) helps us to quantify the connectedness between two variables. So this means they may be somewhat random if taken alone, but taken as pairs, they have something to do with each other.

## The whole semester is bringing us here!

The whole semester has been a buildup to this idea, and we've been talking about it all along. These last three discussions should hopefully seal these ideas firmly into your mind.

## ?!

Are two things connected or aren't they?

## Data type gets in the way

Since data can take different forms, this same fundamental question appears to take various forms, so we need to weild a variety of tools to answer still, that same fundamental question.

## Examples

- Numerical vs. Numerical (LSR)
- Numerical vs. Categorical (ANOVA and t-tests)
- Categorical vs. Categorical (Chi-Square)
- ... and many more we haven't discussed!


## A progression of connectedness

Simple linear regression results:
Dependent Variable: Y_0
Independent Variable: $\overline{\mathrm{X}}$
Y_0 = 118.17895-0.17108076 X
Sample size: 100
$R($ correlation coefficient $)=-0.15690574$
R-sq $=0.024619411$
Estimate of error standard deviation: 16.141497
Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | ---: | ---: | ---: |
| Intercept | 118.17895 | 10.967153 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | -0.17108076 | 0.10877682 | $\neq 0$ | 98 | -1.5727685 | 0.119 |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | ---: |
| Model | 1 | 644.49158 | 644.49158 | 2.4736009 | 0.119 |
| Error | 98 | 25533.697 | 260.54793 |  |  |
| Total | 99 | 26178.189 |  |  |  |



## What I'm doing

With each successive slide, you will see the connectedness increasing.
As we progress, I keep giving the $Y$ variables more of the $X$ value.
Example:

$$
Y_{-} 3=0.7 \cdot Y_{-} 0+0.3 \cdot X
$$

Ironically, the first slide in the sequence shows variables which have a negative correlation. So numerically, the least correlation happens in the second slide after I started giving the response variable some of the explanatory variable!

## What to watch for

- Horizontal line turns into $y=x$ line.
- Points start to hug the LSR line.
- $R^{2}$ increases to $100 \%$
- $p$ - value of the slope increases
- The intercept starts near the average y-value (100).
- The intercept goes to zero progressively.
- The standard error of the intercepts decreases.


## A progression of connectedness - 1

Simple linear regression results:
Dependent Variable: Y_0
Independent Variable: $\overline{\mathrm{X}}$
Y_0 = $118.17895-0.17108076 \mathrm{X}$
Sample size: 100
$R$ (correlation coefficient) $=-0.15690574$
R-sq $=0.024619411$
Estimate of error standard deviation: 16.141497
Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | ---: | ---: | ---: |
| Intercept | 118.17895 | 10.967153 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
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Analysis of variance table for regression model:

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| :--- | ---: | :---: | :---: | :---: | ---: |
| Model | 1 | 644.49158 | 644.49158 | 2.4736009 | 0.119 |
| Error | 98 | 25533.697 | 260.54793 |  |  |
| Total | 99 | 26178.189 |  |  |  |



## A progression of connectedness - 2

## Simple linear regression results:

Dependent Variable: Y_1
Independent Variable: X
$\mathrm{Y}_{-} 1=106.36105-0.05397268 \mathrm{X}$
Sample size: 100
$R$ (correlation coefficient) $=-0.055604464$
R-sq $=0.0030918565$
Estimate of error standard deviation: 14.527347
Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | ---: | ---: | ---: |
| Intercept | 106.36105 | 9.8704377 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | -0.05397268 | 0.097899136 | $\neq 0$ | 98 | -0.55130906 | 0.5827 |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | ---: |
| Model | 1 | 64.145013 | 64.145013 | 0.30394168 | 0.5827 |
| Error | 98 | 20682.295 | 211.04382 |  |  |
| Total | 99 | 20746.44 |  |  |  |



## A progression of connectedness - 3

Simple linear regression results:
Dependent Variable: Y_2
Independent Variable: X
$\mathrm{Y}_{-} 2=94.543159+0.063135395 \mathrm{X}$
Sample size: 100
R (correlation coefficient) $=0.07309208$
$\mathrm{R}-\mathrm{sq}=0.0053424522$
Estimate of error standard deviation: 12.913198

## Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | ---: | ---: | ---: |
| Intercept | 94.543159 | 8.7737224 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | 0.063135395 | 0.087021454 | $\neq 0$ | 98 | 0.72551529 | 0.4699 |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | ---: |
| Model | 1 | 87.772959 | 87.772959 | 0.52637244 | 0.4699 |
| Error | 98 | 16341.566 | 166.75067 |  |  |
| Total | 99 | 16429.339 |  |  |  |



## A progression of connectedness - 4

## Simple linear regression results:

Dependent Variable: Y_3
Independent Variable: X
$\mathrm{Y} \_3=82.725264+0.18024347 \mathrm{X}$
Sample size: 100
R (correlation coefficient) $=0.2325617$
R -sq $=0.054084942$
Estimate of error standard deviation: 11.299048
Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | ---: | ---: | ---: |
| Intercept | 82.725264 | 7.6770071 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | 0.18024347 | 0.076143772 | $\neq 0$ | 98 | 2.3671466 | 0.0199 |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | :---: |
| Model | 1 | 715.37542 | 715.37542 | 5.603383 | 0.0199 |
| Error | 98 | 12511.512 | 127.66848 |  |  |
| Total | 99 | 13226.887 |  |  |  |



## A progression of connectedness - 5

Simple linear regression results:
Dependent Variable: Y_4
Independent Variable: $\bar{X}$
Y_4 = 70.907369 + 0.29735155 X
Sample size: 100
R (correlation coefficient) $=0.41807379$
R-sq = 0.17478569
Estimate of error standard deviation: 9.6848983
Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | ---: | ---: | :--- |
| Intercept | 70.907369 | 6.5802918 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | 0.29735155 | 0.06526609 | $\neq 0$ | 98 | 4.5559883 | $<0.0001$ |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | :---: |
| Model | 1 | 1946.9524 | 1946.9524 | 20.75703 | $<0.0001$ |
| Error | 98 | 9192.1309 | 93.797254 |  |  |
| Total | 99 | 11139.083 |  |  |  |



## A progression of connectedness - 6

Simple linear regression results:
Dependent Variable: Y_5
Independent Variable: X
$\mathrm{Y} \_5=59.089475+0.41445962 \mathrm{X}$
Sample size: 100
$R$ (correlation coefficient) $=0.60998081$
R-sq $=0.37207659$
Estimate of error standard deviation: 8.0707485

## Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | :---: | :---: | :---: |
| Intercept | 59.089475 | 5.4835765 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | 0.41445962 | 0.054388409 | $\neq 0$ | 98 | 7.6203668 | $<0.0001$ |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | :---: |
| Model | 1 | 3782.5039 | 3782.5039 | 58.06999 | $<0.0001$ |
| Error | 98 | 6383.4242 | 65.136982 |  |  |
| Total | 99 | 10165.928 |  |  |  |



## A progression of connectedness - 7

## Simple linear regression results:

Dependent Variable: Y_6
Independent Variable: X
$Y_{-} 6=47.27158+0.5315677 \mathrm{X}$
Sample size: 100
$R$ (correlation coefficient) $=0.77694635$
R-sq $=0.60364563$
Estimate of error standard deviation: 6.4565988
Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | ---: | :---: | :---: |
| Intercept | 47.27158 | 4.3868612 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | 0.5315677 | 0.043510727 | $\neq 0$ | 98 | 12.216934 | $<0.0001$ |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | :---: |
| Model | 1 | 6222.0299 | 6222.0299 | 149.25349 | $<0.0001$ |
| Error | 98 | 4085.3915 | 41.687669 |  |  |
| Total | 99 | 10307.421 |  |  |  |



## A progression of connectedness - 8

## Simple linear regression results:

Dependent Variable: Y 7
Independent Variable: $\overline{\mathrm{X}}$
$Y_{-} 7=35.453685+0.64867577 \mathrm{X}$
Sample size: 100
$R$ (correlation coefficient) $=0.89513658$
R-sq $=0.8012695$
Estimate of error standard deviation: 4.8424491
Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | ---: | :---: | :---: |
| Intercept | 35.453685 | 3.2901459 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | 0.64867577 | 0.032633045 | $\neq 0$ | 98 | 19.877881 | $<0.0001$ |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | :---: |
| Model | 1 | 9265.5304 | 9265.5304 | 395.13013 | $<0.0001$ |
| Error | 98 | 2298.0327 | 23.449314 |  |  |
| Total | 99 | 11563.563 |  |  |  |



## A progression of connectedness - 9

Simple linear regression results:
Dependent Variable: Y _ 8
Independent Variable: $\overline{\mathrm{X}}$
$Y_{-} 8=23.63579+0.76578385 \mathrm{X}$
Sample size: 100
$R$ (correlation coefficient) $=0.96265408$
$\mathrm{R}-\mathrm{sq}=0.92670289$
Estimate of error standard deviation: 3.2282994
Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | ---: | ---: | :---: |
| Intercept | 23.63579 | 2.1934306 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | 0.76578385 | 0.021755363 | $\neq 0$ | 98 | 35.199773 | $<0.0001$ |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | :---: |
| Model | 1 | 12913.005 | 12913.005 | 1239.024 | $<0.0001$ |
| Error | 98 | 1021.3479 | 10.421917 |  |  |
| Total | 99 | 13934.353 |  |  |  |



## A progression of connectedness - 10

## Simple linear regression results:

Dependent Variable: Y_9
Independent Variable: X
$\mathrm{Y} \_9=11.817895+0.88289192 \mathrm{X}$
Sample size: 100
$\mathrm{R}($ correlation coefficient $)=0.99264401$
R -sq $=0.98534213$
Estimate of error standard deviation: 1.6141497
Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | :---: | :---: | :---: |
| Intercept | 11.817895 | 1.0967153 | $\neq 0$ | 98 | 10.775718 | $<0.0001$ |
| Slope | 0.88289192 | 0.010877682 | $\neq 0$ | 98 | 81.165449 | $<0.0001$ |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | :---: | :---: | :---: | :---: |
| Model | 1 | 17164.455 | 17164.455 | 6587.8301 | $<0.0001$ |
| Error | 98 | 255.33697 | 2.6054793 |  |  |
| Total | 99 | 17419.792 |  |  |  |



## A progression of connectedness - 11

Simple linear regression results:
Dependent Variable: Y_10
Independent Variable: $\overline{\mathrm{X}}$
Y $10=0+1 \mathrm{X}$
Sample size: 100
R (correlation coefficient) $=1$
$\mathrm{R}-\mathrm{sq}=1$
Estimate of error standard deviation: 0

## Parameter estimates:

Parameter Estimate Std. Err. Alternative DF T-Stat P-value

| Intercept | 0 | 0 | $\neq 0$ | 98 | NaN | NaN |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Slope | 1 | 0 | $\neq 0$ | 98 | Infinity | NaN |

Analysis of variance table for regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Model | 1 | 22019.879 | 22019.879 | Infinity | $<0.0001$ |
| Error | 98 | 0 | 0 |  |  |
| Total | 99 | 22019.879 |  |  |  |



The reason we approach the $y=x$ line is that both data sets were originally from the same population, and the points themselves were approaching the $y=x$ line. Any perfectly correlated points would lie on some line, but not necessarily a $y=x$ line.

## What does a hypothesis test do?

So, what does this have to do with hypothesis testing?
My goal for you in this course is to see the connections between the concepts, not to just see the course contents as stand-alone tricks you can do to data.

Hypothesis testing in this course is to ask the question of whether two variables are connected or disconnected.

## Example

In the progression we just saw, the initial slide showed randomly generated points which had no underlying reason to be correlated, yet they were negatively correlated!


But the $p$-value for the slope correctly warned us not to be too confident in making a statement about their connectedness! If we'd had a hypothesis that these variables were connected, the $p$-value would have kept us from making a false statement.

## But...

But, sadly, once I started mixing the variables - and I did indeed do this the $p$ - value STILL told me to be cautious, because the sample I had did not provide enough evidence that this mixing was taking place!


I only know it was happening, because I was the one doing it!

## Finally

But eventually, the mixing became apparent!


We could have hypothesized that these variables were connected. We would have said they were, and we'd have been right.

## What does the $p-$ value do?

## The $p$ - value answers a conditional probability question.

If the two variables (or sets of various variables) are not connected, what is the likelihood that we could see a sample with this amount of correlation in it, just because we randomly drew it out that way?

Note: This ignores many important considerations like poorly designed studies and other issues with experimental design and only focuses on the pure theoretical question about sampling errors!

Then, this gives us a $p$-value or perhaps a confidence interval that seeks to answer our conditional probablity question.

## We wish we could ask...

What it doesn't give us, but what we of course wish we could ask, is "What's the probability that there is a connection between these variables?"

Sadly, we can never answer this question, except in quite rare situations.

## Lines today, other things later

This discussion is about Numerical vs. Numerical data. But the same type of discussion will surround ANOVA, t-tests, Chi-Square and many other similar tests which seek to answer the same fundamental question about connectedness.

## Galton Families Data

The data sets in the worksheets are about SAT scores and college GPA, as well as High School GPA. But the data set for this discussion will be a famous data set from Francis Galton (1886).
https://vincentarelbundock.github.io/Rdatasets/doc/ HistData/GaltonFamilies.html

## Sir Francis Galton (16 February 1822 - 17 January 1911)



Galton produced over 340 papers and books. He also created the statistical concept of correlation and widely promoted regression toward the mean. (Wikipedia)

## Data Overview

## GaltonFamilies.csv

StatCrunch * Applets * Edit * Data * Stat * Graph *

| Row | family | father | mother | midparentHeigl | children | childNum | gender | childHeight |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 78.5 | 67 | 75.43 | 4 | 1 | male | 73.2 |
| 2 | 1 | 78.5 | 67 | 75.43 | 4 | 2 | female | 69.2 |
| 3 | 1 | 78.5 | 67 | 75.43 | 4 | 3 | female | 69 |
| 4 | 1 | 78.5 | 67 | 75.43 | 4 | 4 | female | 69 |
| 5 | 2 | 75.5 | 66.5 | 73.66 | 4 | 1 | male | 73.5 |
| 6 | 2 | 75.5 | 66.5 | 73.66 | 4 | 2 | male | 72.5 |
| 7 | 2 | 75.5 | 66.5 | 73.66 | 4 | 3 | female | 65.5 |
| 8 | 2 | 75.5 | 66.5 | 73.66 | 4 | 4 | female | 65.5 |
| 9 | 3 | 75 | 64 | 72.06 | 2 | 1 | male | 71 |
| 10 | 3 | 75 | 64 | 72.06 | 2 | 2 | female | 68 |
| 11 | 4 | 75 | 64 | 72.06 | 5 | 1 | male | 70.5 |
| 12 | 4 | 75 | 64 | 72.06 | 5 | 2 | male | 68.5 |
| 13 | 4 | 75 | 64 | 72.06 | 5 | 3 | female | 67 |
| 14 | 4 | 75 | 64 | 72.06 | 5 | 4 | female | 64.5 |
| 15 | 4 | 75 | 64 | 72.06 | 5 | 5 | female | 63 |
| 16 | 5 | 75 | 58.5 | 69.09 | 6 | 1 | male | 72 |
| 17 | 5 | 75 | 58.5 | 69.09 | 6 | 2 | male | 69 |
| 18 | 5 | 75 | 58.5 | 69.09 | 6 | 3 | male | 68 |
| 19 | 5 | 75 | 58.5 | 69.09 | 6 | 4 | female | 66.5 |
| 20 | 5 | 75 | 58.5 | 69.09 | 6 | 5 | female | 62.5 |
| 21 | 5 | 75 | 58.5 | 69.09 | 6 | 6 | female | 62.5 |

We are looking at some families and their heights with gender. Birth order is not given, although it appears to be. It's not.

## Scatterplot by gender



The average of the two parents' heights is the explanatory variable, and the (adult) child's height is the response.

## Some results

## male children

$$
\text { child }=20+0.71 \text { parent }
$$

## female children

$$
\text { child }=18+0.66 \text { parent }
$$

Note here how ridiculous the intercepts are, and how utterly meaningless. This says that if a parent is zero inches tall, the male children will end up 2 inches taller than their sisters.

Really, if you look at the range of the data, the difference between the male and female children is between 5.2 and 5.8 inches.

## To simplify

To simplify, we don't have to group by gender.

$$
\text { child }=22.6+0.637 \text { parent }
$$

Where, again, the parent height means the average of the two parents.

## Can we find the LSR line?

There is a process we are not covering in this class, and it feels the same as finding a standard deviation. There are lots of subtractions, squaring, and square rooting.

But if the standard deviations for each variable individually as well as the correlation between the two variables has already been calculated, you can use those values to find the LSR line!

## Formula time

$$
\begin{aligned}
& y=a+b x \\
& \text { if } \quad b=r \frac{s_{y}}{s_{x}} \\
& \text { and } \quad a=\bar{y}-b \cdot \bar{x}
\end{aligned}
$$

and $r$ is the correlation between $x$ and $y$, with $s_{x}$ and $s_{y}$ being the sample standard deviations and $\bar{x}$ and $\bar{y}$ being the means.

## Let's try it!

Simple linear regression results:
Dependent Variable: childHeight
Independent Variable: midparentHeight
childHeight $=22.636241+0.6373609$ midparentHeight
Sample size: 934
R (correlation coefficient) $=0.3209499$
R -sq $=0.10300884$
Estimate of error standard deviation: 3.3917132

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
| :--- | :---: | :---: | ---: | :---: | :---: | :---: |
| Intercept | 22.636241 | 4.2651074 | $\neq 0$ | 932 | 5.3073084 | $<0.0001$ |
| Slope | 0.6373609 | 0.061607602 | $\neq 0$ | 932 | 10.345491 | $<0.0001$ |

We need $r, \bar{x}, \bar{y}, s_{x}$, and $s_{y}$.
Let's use $0.321,69.2,66.75,1.80$, and 3.58 .

$$
\begin{gathered}
b=0.321 \frac{3.58}{1.80}=0.638 \\
a=66.75-0.638 \cdot 69.21=22.6
\end{gathered}
$$

Which agrees fairly well with child $=22.6+0.637$ parent from StatCrunch!

## Standard Error

Simple linear regression results:
Dependent Variable: childHeight
Independent Variable: midparentHeight
childHeight $=22.636241+0.6373609$ midparentHeight Sample size: 934
$R$ (correlation coefficient) $=0,3209499$
$\mathrm{R}-\mathrm{sq}=0.10300884$
Estimate of error standard deviation: 3.3917132

| Intercept | 22.636241 | 4.2651074 | $\neq 0$ | 932 | 5.3073084 | $<0.0001$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Slope | 0.6373609 | 0.061607602 | $\neq 0$ | 932 | 10.345491 | $<0.0001$ |

Note: Standard Error is another term for the standard deviation of the sampling distribution.

$$
\begin{aligned}
& s_{x}=1.80237 \text { so } S E_{x}=1.80237 / \sqrt{934}=0.058975 \\
& s_{y}=3.57925 \text { so } S E_{y}=3.5925 / \sqrt{934}=0.117117
\end{aligned}
$$

Why is $s_{x}$ so much lower than $s_{y}$ ?
It's the average of the two parents' heights, so it's going to be roughly a factor of $\sqrt{2}$ less than the child's height. (It's actually even less than that though.)

## Adding variables that are correlated

Let's try adding the parent height and the child height, which are correlated, just to get some experience adding correlated variables. (On the worksheet, you will add SAT math and SAT verbal scores.)

Data $>$ Compute $>$ Expression and I've called this 'sum'.
Let's see if this formula works!

$$
S_{A+B}^{2}=S_{A}^{2}+S_{B}^{2}+2 r S_{A} S_{B}
$$

Remember, of course, if $r=0$ you get back to the formula you already know how to use!

## Testing our new formula

## Summary statistics:

| Column $\boldsymbol{c \|}$ | Mean $\uparrow$ | Variance $\boldsymbol{*}$ | Std. dev. $\bullet$ |
| :--- | ---: | ---: | ---: |
| midparentHeight | 69.206773 | 3.2485384 | 1.8023702 |
| childHeight | 66.745931 | 12.811039 | 3.5792512 |
| sum | 135.9527 | 20.20056 | 4.4945033 |

Correlation between childHeight and midparentHeight is: 0.3209499

$$
\begin{gathered}
S_{A+B}^{2} \stackrel{?}{=} S_{A}^{2}+S_{B}^{2}+2 r S_{A} S_{B} \\
20.20 \stackrel{?}{=} 3.25+12.8+2(0.321)(1.80)(3.58) \\
20.20 \stackrel{?}{=} 16.05+4.14 \\
20.20 \approx 20.19
\end{gathered}
$$

Close enough! (It's unequal due only to roundoff error.)

## Multi-linear models

StatCrunch makes it easy to use multi-linear models. This allows us to use the fathers' and mothers' heights as two different inputs to our model. We can find out whose genes matter more.

$$
\text { Stat }>\text { Regression }>\text { Multiple Linear }
$$

## Dad matters more!

Multiple linear regression results:
Dependent Variable: childHeight
Independent Variable(s): father, mother
childHeight $=22.64328+0.36828233$ father +0.29050997 mother
Parameter estimates:

| Parameter * | Estimate $\uparrow$ | Std. Err. * | Alternative $*$ | DF * | T-Stat * | P-value * |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 22.64328 | 4.2621271 | $\neq 0$ | 931 | 5.3126712 | $<0.0001$ |
| father | 0.36828233 | 0.044888233 | $\neq 0$ | 931 | 8.2044293 | $<0.0001$ |
| mother | 0.29050997 | 0.048524794 | $\neq 0$ | 931 | 5.9868359 | $<0.0001$ |

Analysis of variance table for multiple regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Model | 2 | 1257.7124 | 628.8562 | 54.742013 | $<0.0001$ |
| Error | 931 | 10694.987 | 11.487634 |  |  |
| Total | 933 | 11952.7 |  |  |  |

Summary of fit:
Root MSE: 3.3893412
R-squared: 0.1052
R-squared (adjusted): 0.1033

$$
R^{2}=10.5 \%
$$

So, this is a little bit better than without splitting the parents up. But, we only gain about half a percent on $R^{2}$.

## Double split!

Let's split the entire problem by gender. We can split that in the model by using the father and mother separately, and we can split on the gender of the child as well.

Multiple linear regression results for gender=male:
Dependent Variable: childHeight
Independent Variable(s): father, mother
childHeight $=19.312813+0.41755622$ father +0.32877342 mother
Parameter estimates:

| Parameter | Estimate $\phi$ | Std. Err. $\bullet$ | Alternative $\phi$ | DF $\theta$ | T-Stat $\phi$ | P-value $\bullet$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | 19.312813 | 4.0950285 | $\neq 0$ | 478 | 4.7161609 | $<0.0001$ |
| father | 0.41755622 | 0.045612392 | $\neq 0$ | 478 | 9.1544468 | $<0.0001$ |
| mother | 0.32877342 | 0.04530087 | $\neq 0$ | 478 | 7.257552 | $<0.0001$ |

Analysis of variance table for multiple regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | :--- | :--- | :--- | :--- | :--- |



| Error | 478 | 2518.4162 | 5.2686532 |
| :--- | :--- | :--- | :--- | :--- |


| Total | 480 | 3304.7408 |
| :--- | :--- | :--- | :--- |

Summary of fit:
Root MSE: 2.2953547
R-squared: 0.2379
R-squared (adjusted); 0.2347

Multiple linear regression results for gender=female:
Dependent Variable: childHeight
independent Variable(s): father, mother
childHeight $=18.833583+0.37254233$ father +0.30348214 mother
Parameter estimates:

| Parameter * | Estimate $\dagger$ | Std. Err. ${ }^{\text {e }}$ | Alternative $\dagger$ | DF 0 | T-Stat * | $\mathbf{P}$-value ${ }^{\text {¢ }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 18.833583 | 3.6094745 | $\neq 0$ | 450 | 5.2178184 | $<0.0001$ |
| father | 0.37254233 | 0.035912028 | $\neq 0$ | 450 | 10.373748 | $<0.0001$ |
| mother | 0.30348214 | 0.042080619 | $\neq 0$ | 450 | 7.2119219 | $<0.0001$ |

Analysis of variance table for multiple regression model:

| Source | DF | SS | MS | F-stat | P-value |
| :--- | ---: | ---: | :---: | :---: | :---: |
| Model | 2 | 672.557 | 336.2785 | 82.437552 | $<0.0001$ |
| Error | 450 | 1835.6358 | 4.0791908 |  |  |
| Total | 452 | 2508.1928 |  |  |  |

Summary of fit:
Root MSE: 2.0197007
R-squared: 0.2681
R-squared (adjusted): 0.2649

For males $R^{2}=23.8 \%$, and for females $R^{2}=26.8 \%$.

## Splitting the parents didn't help much

With splitting: for males $R^{2}=23.8 \%$, and for females $R^{2}=26.8 \%$. Without splitting: for males $R^{2}=23.3 \%$, and for females $R^{2}=26.3 \%$.

So, we didn't gain much with all that effort. However, it was just clicking an option on the computer, so it really wasn't much effort anyhow.

## Worksheet time!

Go have fun!

## MEMORY QUESTIONs




You calculate the best fit line for a collection of data, and the line you plot looks very horizontal to you when you look at the computer screen. Do you suspect a high or low correlation for that data? Why?

High correlations would give us lines sloped up or sloped down.

Very high.

Flat lines mean high correlations.

## Very low.

## SUBMIT




You calculate the best fit line for a collection of data, and the line you plot looks very horizontal to you when you look at the computer screen. Do you suspect a high or low correlation for that data? Why?

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Very high.

Flat lines mean high correlations.

## Very low.

## SUBMIT

## STAT 202 Memory Questions

## Combined Sets

To sign the log and earn credit, you need to work the combined set. You are allowed a maximum of 7 errors. You need to get 50 right in 13 minutes.

Click all correct answers, then click submit:


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## STAT 202 Memory Questions

Combined Sets
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Click all correct answers, then click submit:


## STAT 202 Memory Questions

```
Combined Sets
```

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