When variances don't add

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Least Squares Regression (LSR, or a line of best fit) helps us to quantify the connectedness between two variables. So this means they may be somewhat random if taken alone, but taken as pairs, they have something to do with each other. The whole semester has been a buildup to this idea, and we've been talking about it all along. These last three discussions should hopefully seal these ideas firmly into your mind.

?!

Are two things connected or aren't they?

Since data can take different forms, this same fundamental question appears to take various forms, so we need to weild a variety of tools to answer still, that same fundamental question.

- Numerical vs. Numerical (LSR)
- Numerical vs. Categorical (ANOVA and t-tests)
- Categorical vs. Categorical (Chi-Square)
- ... and many more we haven't discussed!

 $\begin{array}{l} \label{eq:stability} \textbf{Simple linear regression results:} \\ Dependent Variable: X_0 \\ Independent Variable: X \\ Y_0 = 118.17895 - 0.17108076 X \\ Sample size: 100 \\ R (correlation coefficient) = -0.15690574 \\ R \cdot sq = 0.024619411 \\ Estimate 0 ferror standard deviation: 16.141497 \\ \end{array}$

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value | |
|-----------|---------------------|-----------|-------------|----|------------|----------|--|
| Intercept | Intercept 118.17895 | | ≠ 0 | 98 | 10.775718 | < 0.0001 | |
| Slope | Slope -0.17108076 | | ≠ 0 | 98 | -1.5727685 | 0.119 | |

| Source | DF SS | | MS | F-stat | P-value |
|--------|-------|-----------|-----------|-----------|---------|
| Model | 1 | 644.49158 | 644.49158 | 2.4736009 | 0.119 |
| Error | 98 | 25533.697 | 260.54793 | | |
| Total | 99 | 26178.189 | | | |



With each successive slide, you will see the connectedness increasing. As we progress, I keep giving the Y variables more of the X value. Example:

$$Y_{-3} = 0.7 \cdot Y_{-0} + 0.3 \cdot X$$

Ironically, the first slide in the sequence shows variables which have a negative correlation. So numerically, the least correlation happens in the second slide after I started giving the response variable some of the explanatory variable!

- Horizontal line turns into y = x line.
- Points start to hug the LSR line.
- R² increases to 100%
- *p value* of the slope increases
- The intercept starts near the average y-value (100).
- The intercept goes to zero progressively.
- The standard error of the intercepts decreases.

 $\begin{array}{l} \label{eq:stability} \textbf{Simple linear regression results:} \\ Dependent Variable: X_0 \\ Independent Variable: X \\ Y_0 = 118.17895 - 0.17108076 X \\ Sample size: 100 \\ R (correlation coefficient) = -0.15690574 \\ R \cdot sq = 0.024619411 \\ Estimate 0 ferror standard deviation: 16.141497 \\ \end{array}$

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| Total | 99 | 26178.189 | | | |



Simple linear regression results: Dependent Variable: Y_1 Independent Variable: X Y_1 = 106.36105 - 0.05397268 X Sample size: 100 R (correlation coefficient) = -0.055604464 R-sq = 0.0030918565 Estimate of error standard deviation: 14.527347

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|-------------|-------------|-------------|----|-------------|---------|
| Intercept | 106.36105 | 9.8704377 | ≠ 0 | 98 | 10.775718 | <0.0001 |
| Slope | -0.05397268 | 0.097899136 | ≠ 0 | 98 | -0.55130906 | 0.5827 |

| Source | DF | SS | MS | F-stat | P-value | |
|--------|----|-----------|-----------|------------|---------|--|
| Model | 1 | 64.145013 | 64.145013 | 0.30394168 | 0.5827 | |
| Error | 98 | 20682.295 | 211.04382 | | | |
| Total | 99 | 20746.44 | | | | |



Simple linear regression results: Dependent Variable: Y_2 Independent Variable: X $Y_2 = 94,543159 + 0.063135395 X$ Sample size: 100 R (correlation coefficient) = 0.07309208 R-sq = 0.0053424522 Estimate of error standard deviation: 12.913198

Parameter estimates:

| Parameter | arameter Estimate | | Alternative | DF | T-Stat | P-value |
|-----------|-------------------|-----------|-------------|----|------------|----------|
| Intercept | 94.543159 | 8.7737224 | ≠ 0 | 98 | 10.775718 | < 0.0001 |
| Slope | Slope 0.063135395 | | ≠ 0 | 98 | 0.72551529 | 0.4699 |

| Source | DF | SS | MS | F-stat | P-value |
|--------|----|-----------|-----------|------------|---------|
| Model | 1 | 87.772959 | 87.772959 | 0.52637244 | 0.4699 |
| Error | 98 | 16341.566 | 166.75067 | | |
| Total | 99 | 16429.339 | | | |



Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|------------|-------------|-------------|----|-----------|----------|
| Intercept | 82.725264 | 7.6770071 | ≠ 0 | 98 | 10.775718 | < 0.0001 |
| Slope | 0.18024347 | 0.076143772 | ≠ 0 | 98 | 2.3671466 | 0.0199 |

| Source | DF | SS | MS | F-stat | P-value |
|--------|----|-----------|-----------|----------|---------|
| Model | 1 | 715.37542 | 715.37542 | 5.603383 | 0.0199 |
| Error | 98 | 12511.512 | 127.66848 | | |
| Total | 99 | 13226.887 | | | |



| Simple linear regression results: |
|---|
| Dependent Variable: Y_4 |
| Independent Variable: X |
| Y_4 = 70.907369 + 0.29735155 X |
| Sample size: 100 |
| R (correlation coefficient) = 0.41807379 |
| R-sq = 0.17478569 |
| Estimate of error standard deviation: 9.6848983 |

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|------------|------------|-------------|----|-----------|----------|
| Intercept | 70.907369 | 6.5802918 | ≠ 0 | 98 | 10.775718 | < 0.0001 |
| Slope | 0.29735155 | 0.06526609 | ≠ 0 | 98 | 4.5559883 | < 0.0001 |

| Source | DF | SS | MS | F-stat | P-value |
|--------|-----|-----------|-----------|----------|---------|
| Model | - 1 | 1946.9524 | 1946.9524 | 20.75703 | <0.0001 |
| Error | 98 | 9192.1309 | 93.797254 | | |
| Total | 99 | 11139.083 | | | |



$$\begin{split} & \text{Simple linear regression results:} \\ & \text{Dependent Variable: } X \\ & \text{Independent Variable: } X \\ & \text{Y_5} = 59.089475 + 0.41445962 X \\ & \text{Sample size: 100} \\ & \text{R (correlation coefficient)} = 0.60998081 \\ & \text{R-sq} = 0.37207659 \\ & \text{Estimate of error standard deviation: } 8.0707485 \end{split}$$

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|------------|-------------|-------------|----|-----------|----------|
| Intercept | 59.089475 | 5.4835765 | ≠ 0 | 98 | 10.775718 | < 0.0001 |
| Slope | 0.41445962 | 0.054388409 | ≠ 0 | 98 | 7.6203668 | < 0.0001 |

| Source | DF | SS | MS | F-stat | P-value |
|--------|----|-----------|-----------|----------|----------|
| Model | 1 | 3782.5039 | 3782.5039 | 58.06999 | < 0.0001 |
| Error | 98 | 6383.4242 | 65.136982 | | |
| Total | 99 | 10165.928 | | | |



| Simple linear regression results: |
|---|
| Dependent Variable: Y_6 |
| Independent Variable: X |
| Y_6 = 47.27158 + 0.5315677 X |
| Sample size: 100 |
| R (correlation coefficient) = 0.77694635 |
| R-sq = 0.60364563 |
| Estimate of error standard deviation: 6.4565988 |

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|-----------|-------------|-------------|----|-----------|----------|
| Intercept | 47.27158 | 4.3868612 | ≠ 0 | 98 | 10.775718 | < 0.0001 |
| Slope | 0.5315677 | 0.043510727 | ≠ 0 | 98 | 12.216934 | < 0.0001 |

| Source | DF | SS | MS | F-stat | P-value |
|--------|----|-----------|-----------|-----------|----------|
| Model | 1 | 6222.0299 | 6222.0299 | 149.25349 | < 0.0001 |
| Error | 98 | 4085.3915 | 41.687669 | | |
| Total | 99 | 10307.421 | | | |



Simple linear regression results: Dependent Variable: Y_{-7} Independent Variable: X $Y_{-7} = 33.453685 + 0.64867577 X$ Sample size: 100 R (correlation coefficient) = 0.89513658 R-sq = 0.8012695 Estimate of error standard deviation: 4.8424491

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|------------|-------------|-------------|----|-----------|----------|
| Intercept | 35.453685 | 3.2901459 | ≠ 0 | 98 | 10.775718 | < 0.0001 |
| Slope | 0.64867577 | 0.032633045 | ≠ 0 | 98 | 19.877881 | < 0.0001 |

| Source | DF | SS | MS | F-stat | P-value |
|--------|----|-----------|-----------|-----------|----------|
| Model | 1 | 9265.5304 | 9265.5304 | 395.13013 | < 0.0001 |
| Error | 98 | 2298.0327 | 23.449314 | | |
| Total | 99 | 11563.563 | | | |



| Simple linear regression results: |
|---|
| Dependent Variable: Y_8 |
| Independent Variable: X |
| Y_8 = 23.63579 + 0.76578385 X |
| Sample size: 100 |
| R (correlation coefficient) = 0.96265408 |
| R-sq = 0.92670289 |
| Estimate of error standard deviation: 3.2282994 |

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|------------|-------------|-------------|----|-----------|----------|
| Intercept | 23.63579 | 2.1934306 | ≠ 0 | 98 | 10.775718 | < 0.0001 |
| Slope | 0.76578385 | 0.021755363 | ≠ 0 | 98 | 35.199773 | < 0.0001 |

| Source | DF | SS | MS | F-stat | P-value |
|--------|----|-----------|-----------|----------|----------|
| Model | 1 | 12913.005 | 12913.005 | 1239.024 | < 0.0001 |
| Error | 98 | 1021.3479 | 10.421917 | | |
| Total | 99 | 13934.353 | | | |



 $\begin{array}{l} \textbf{Simple linear regression results:} \\ Dependent Variable: Y_9 \\ Independent Variable: X \\ Y_9 = 11.817895 + 0.88289192 X \\ Sample size: 100 \\ R(correlation coefficient) = 0.99264401 \\ R-sq = 0.98533213 \\ Estimate of error standard deviation: 1.6141497 \\ \end{array}$

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|------------|-------------|-------------|----|-----------|----------|
| Intercept | 11.817895 | 1.0967153 | ≠ 0 | 98 | 10.775718 | <0.0001 |
| Slope | 0.88289192 | 0.010877682 | ≠0 | 98 | 81.165449 | < 0.0001 |

| Source | DF | SS | MS | F-stat | P-value |
|--------|----|-----------|-----------|-----------|----------|
| Model | 1 | 17164.455 | 17164.455 | 6587.8301 | < 0.0001 |
| Error | 98 | 255.33697 | 2.6054793 | | |
| Total | 99 | 17419.792 | | | |



$$\begin{split} & \text{Simple linear regression results:} \\ & \text{Dependent Variable: } Y_{-10} \\ & \text{Independent Variable: } X \\ & Y_{-10} = 0 + 1. X \\ & \text{Sample size: } 100 \\ & \text{R (correlation coefficient)} = 1 \\ & \text{R-sq} = 1 \\ & \text{Estimate of error standard deviation: } 0 \end{split}$$

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|----------|-----------|-------------|----|----------|---------|
| Intercept | 0 | 0 | ≠ 0 | 98 | NaN | NaN |
| Slope | 1 | 0 | ≠ 0 | 98 | Infinity | NaN |

| Source | DF | SS | MS | F-stat | P-value |
|--------|----|-----------|-----------|----------|----------|
| Model | 1 | 22019.879 | 22019.879 | Infinity | < 0.0001 |
| Error | 98 | 0 | 0 | | |
| Total | 99 | 22019.879 | | | |



The reason we approach the y = x line is that both data sets were originally from the same population, and the points themselves were approaching the y = x line. Any perfectly correlated points would lie on some line, but not necessarily a y = x line. So, what does this have to do with hypothesis testing?

My goal for you in this course is to see the connections between the concepts, not to just see the course contents as stand-alone tricks you can do to data.

Hypothesis testing in this course is to ask the question of whether two variables are connected or disconnected.

In the progression we just saw, the initial slide showed randomly generated points which had no underlying reason to be correlated, yet they were negatively correlated!



But the p - value for the slope correctly warned us not to be too confident in making a statement about their connectedness! If we'd had a hypothesis that these variables were connected, the p - value would have kept us from making a false statement. But, sadly, once I started mixing the variables - and I did indeed do this - the p - value STILL told me to be cautious, because the sample I had did not provide enough evidence that this mixing was taking place!



I only know it was happening, because I was the one doing it!

But eventually, the mixing became apparent!



We could have hypothesized that these variables were connected. We would have said they were, and we'd have been right.

The p - value answers a conditional probability question.

If the two variables (or sets of various variables) are **not** connected, what is the likelihood that we could see a sample with this amount of correlation in it, just because we randomly drew it out that way?

Note: This ignores many important considerations like poorly designed studies and other issues with experimental design and only focuses on the pure theoretical question about sampling errors!

Then, this gives us a p - value or perhaps a confidence interval that seeks to answer our conditional probablity question.

What it doesn't give us, but what we of course wish we could ask, is "What's the probability that there **is** a connection between these variables?"

Sadly, we can never answer this question, except in quite rare situations.

This discussion is about Numerical vs. Numerical data. But the same type of discussion will surround ANOVA, t-tests, Chi-Square and many other similar tests which seek to answer the same fundamental question about connectedness.

- The data sets in the worksheets are about SAT scores and college GPA, as well as High School GPA. But the data set for this discussion will be a famous data set from Francis Galton (1886).
- https://vincentarelbundock.github.io/Rdatasets/doc/ HistData/GaltonFamilies.html

Sir Francis Galton (16 February 1822 – 17 January 1911)



Galton produced over 340 papers and books. He also created the statistical concept of correlation and widely promoted regression toward the mean. (Wikipedia)

GaltonFamilies.csv

| StatCrunch - | Applets • | Edit - | Data 🔻 | Stat 🔻 | Graph 🔻 | |
|--------------|-----------|--------|--------|--------|---------|--|
|--------------|-----------|--------|--------|--------|---------|--|

| Row | family | father | mother | midparentHeigl | children | childNum | gender | childHeight |
|-----|--------|--------|--------|----------------|----------|----------|--------|-------------|
| 1 | 1 | 78.5 | 67 | 75.43 | 4 | 1 | male | 73.2 |
| 2 | 1 | 78.5 | 67 | 75.43 | 4 | 2 | female | 69.2 |
| 3 | 1 | 78.5 | 67 | 75.43 | 4 | 3 | female | 69 |
| 4 | 1 | 78.5 | 67 | 75.43 | 4 | 4 | female | 69 |
| 5 | 2 | 75.5 | 66.5 | 73.66 | 4 | 1 | male | 73.5 |
| 6 | 2 | 75.5 | 66.5 | 73.66 | 4 | 2 | male | 72.5 |
| 7 | 2 | 75.5 | 66.5 | 73.66 | 4 | 3 | female | 65.5 |
| 8 | 2 | 75.5 | 66.5 | 73.66 | 4 | 4 | female | 65.5 |
| 9 | 3 | 75 | 64 | 72.06 | 2 | 1 | male | 71 |
| 10 | 3 | 75 | 64 | 72.06 | 2 | 2 | female | 68 |
| 11 | 4 | 75 | 64 | 72.06 | 5 | 1 | male | 70.5 |
| 12 | 4 | 75 | 64 | 72.06 | 5 | 2 | male | 68.5 |
| 13 | 4 | 75 | 64 | 72.06 | 5 | 3 | female | 67 |
| 14 | 4 | 75 | 64 | 72.06 | 5 | 4 | female | 64.5 |
| 15 | 4 | 75 | 64 | 72.06 | 5 | 5 | female | 63 |
| 16 | 5 | 75 | 58.5 | 69.09 | 6 | 1 | male | 72 |
| 17 | 5 | 75 | 58.5 | 69.09 | 6 | 2 | male | 69 |
| 18 | 5 | 75 | 58.5 | 69.09 | 6 | 3 | male | 68 |
| 19 | 5 | 75 | 58.5 | 69.09 | 6 | 4 | female | 66.5 |
| 20 | 5 | 75 | 58.5 | 69.09 | 6 | 5 | female | 62.5 |
| 21 | 5 | 75 | 58.5 | 69.09 | 6 | 6 | female | 62.5 |

We are looking at some families and their heights with gender. Birth order is not given, although it appears to be. It's not.

Scatterplot by gender



The average of the two parents' heights is the explanatory variable, and the (adult) child's height is the response.

male children

$\mathit{child} = 20 + 0.71 \mathit{parent}$

female children

child = 18 + 0.66 parent

Note here how ridiculous the intercepts are, and how utterly meaningless. This says that if a parent is zero inches tall, the male children will end up 2 inches taller than their sisters.

Really, if you look at the range of the data, the difference between the male and female children is between 5.2 and 5.8 inches.

To simplify, we don't have to group by gender.

child = 22.6 + 0.637 parent

Where, again, the parent height means the average of the two parents.

There is a process we are not covering in this class, and it feels the same as finding a standard deviation. There are lots of subtractions, squaring, and square rooting.

But if the standard deviations for each variable individually as well as the correlation between the two variables has already been calculated, you can use those values to find the LSR line!

$$y = a + bx$$

if $b = r \frac{s_y}{s_x}$

and
$$a = \bar{y} - b \cdot \bar{x}$$

and r is the correlation between x and y, with s_x and s_y being the sample standard deviations and \bar{x} and \bar{y} being the means.

Let's try it!

Simple linear regression results: Dependent Variable: childfreight Independent Variable: midparentHeight Sample size: 934 R (correlation coefficient) = 0.3209499 R-sq = 0.10300884 Estimate of error standard deviation: 3.3917132

| Summary | statistics: |
|---------|-------------|
| | |

| Column ¢ | Mean ø | Std. dev. ϕ | Std. err. ø |
|-----------------|-----------|------------------|-------------|
| midparentHeight | 69.206773 | 1.8023702 | 0.058975355 |
| childHeight | 66.745931 | 3.5792512 | 0.11711668 |

Parameter estimates:

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|-----------|-------------|-------------|-----|-----------|----------|
| Intercept | 22.636241 | 4.2651074 | ≠ 0 | 932 | 5.3073084 | < 0.0001 |
| Slope | 0.6373609 | 0.061607602 | ≠ 0 | 932 | 10.345491 | < 0.0001 |

We need r, \bar{x}, \bar{y}, s_x , and s_y . Let's use 0.321, 69.2, 66.75, 1.80, and 3.58.

$$b = 0.321 \frac{3.58}{1.80} = 0.638$$

$$a = 66.75 - 0.638 \cdot 69.21 = 22.6$$

Which agrees fairly well with child = 22.6 + 0.637 parent from StatCrunch!

| Simple linear regression results: | Summary statisti | cs: | | |
|---|------------------|-----------|------------------|-------------|
| Dependent Variable: childHeight | Column ¢ | Mean ¢ | Std. dev. ϕ | Std. err. ¢ |
| Independent Variable: midparentHeight childHeight = 22.636241 + 0.6373609 midparentHeight | midparentHeight | 69.206773 | 1.8023702 | 0.058975355 |
| Sample size: 934 | childHeight | 66.745931 | 3.5792512 | 0.11711668 |
| R (correlation coefficient) = 0.3209499 R-sq = 0.10300884 Estimate of error standard deviation: 3.3917132 | | | | |
| | | | | |

| Parameter | Estimate | Std. Err. | Alternative | DF | T-Stat | P-value |
|-----------|-----------|-------------|-------------|-----|-----------|----------|
| Intercept | 22.636241 | 4.2651074 | ≠ 0 | 932 | 5.3073084 | < 0.0001 |
| Slope | 0.6373609 | 0.061607602 | ≠ 0 | 932 | 10.345491 | < 0.0001 |

Note: **Standard Error** is another term for the standard deviation of the sampling distribution.

$$s_x = 1.80237$$
 so $SE_x = 1.80237/\sqrt{934} = 0.058975$

$$s_v = 3.57925$$
 so $SE_v = 3.5925/\sqrt{934} = 0.117117$

Why is s_x so much lower than s_y ?

It's the average of the two parents' heights, so it's going to be roughly a factor of $\sqrt{2}$ less than the child's height. (It's actually even less than that though.)

Let's try adding the parent height and the child height, which are correlated, just to get some experience adding correlated variables. (On the worksheet, you will add SAT math and SAT verbal scores.)

Data > Compute > Expression and I've called this 'sum'.

Let's see if this formula works!

$$S_{A+B}^2 = S_A^2 + S_B^2 + 2rS_AS_B$$

Remember, of course, if r = 0 you get back to the formula you already know how to use!

Testing our new formula

Summary statistics:

| Column ¢ | Mean ø | Variance ϕ | Std. dev. \$ |
|-----------------|-----------|-----------------|--------------|
| midparentHeight | 69.206773 | 3.2485384 | 1.8023702 |
| childHeight | 66.745931 | 12.811039 | 3.5792512 |
| sum | 135.9527 | 20.20056 | 4.4945033 |

Correlation between childHeight and midparentHeight is: 0.3209499

$$S_{A+B}^2 \stackrel{?}{=} S_A^2 + S_B^2 + 2rS_A S_B$$

$$20.20 \stackrel{?}{=} 3.25 + 12.8 + 2(0.321)(1.80)(3.58)$$

$$20.20 \stackrel{?}{=} 16.05 + 4.14$$

$$20.20 \approx 20.19$$

Close enough! (It's unequal due only to roundoff error.)

StatCrunch makes it easy to use multi-linear models. This allows us to use the fathers' and mothers' heights as two different inputs to our model. We can find out whose genes matter more.

Stat > Regression > Multiple Linear

Multiple linear regression results:

Dependent Variable: childHeight Independent Variable(s): father, mother childHeight = 22.64328 + 0.36828233 father + 0.29050997 mother

Parameter estimates:

| Parameter 🖨 | Estimate 🛊 | Std. Err. \$ | Alternative \$ | DF \$ | T-Stat ¢ | P-value \$ |
|-------------|------------|--------------|----------------|-------|-----------|------------|
| Intercept | 22.64328 | 4.2621271 | ≠ 0 | 931 | 5.3126712 | < 0.0001 |
| father | 0.36828233 | 0.044888233 | ≠ 0 | 931 | 8.2044293 | < 0.0001 |
| mother | 0.29050997 | 0.048524794 | ≠ 0 | 931 | 5.9868359 | < 0.0001 |

Analysis of variance table for multiple regression model:

| Source | DF | SS | MS | F-stat | P-value |
|--------|-----|-----------|-----------|-----------|----------|
| Model | 2 | 1257.7124 | 628.8562 | 54.742013 | < 0.0001 |
| Error | 931 | 10694.987 | 11.487634 | | |
| Total | 933 | 11952.7 | | | |

Summary of fit:

Root MSE: 3.3893412 R-squared: 0.1052 R-squared (adjusted): 0.1033

 $R^2 = 10.5\%$

So, this is a little bit better than without splitting the parents up. But, we only gain about half a percent on R^2 .

Double split!

Let's split the entire problem by gender. We can split that in the model by using the father and mother separately, and we can split on the gender of the child as well.

Multiple linear regression results for gender=male:

Dependent Variable: childHeight Independent Variable(s): father, mother childHeight = 19.312813 + 0.41755622 father + 0.32877342 mother

Parameter estimates:

| Parameter ϕ | Estimate ϕ | Std. Err. ø | Alternative \$ | DF ¢ | T-Stat ¢ | P-value \$ |
|------------------|-----------------|-------------|----------------|------|-----------|------------|
| Intercept | 19.312813 | 4.0950285 | ≠ 0 | 478 | 4.7161609 | <0.0001 |
| father | 0.41755622 | 0.045612392 | ≠ 0 | 478 | 9.1544468 | < 0.0001 |
| mother | 0.32877342 | 0.04530087 | ≠ 0 | 478 | 7.257552 | < 0.0001 |

Analysis of variance table for multiple regression model:

| Source | DF | SS | MS | F-stat | P-value |
|--------|-----|-----------|-----------|-----------|----------|
| Model | 2 | 786.32461 | 393.16231 | 74.622924 | < 0.0001 |
| Error | 478 | 2518.4162 | 5.2686532 | | |
| Total | 480 | 3304.7408 | | | |

Summary of fit: Root MSE: 2.2953547 R-squared: 0.2379 R-squared (adjusted): 0.2347

Multiple linear regression results for gender=female:

Dependent Variable: childHeight Independent Variable(s): father, mother childHeight = 18.833583 + 0.37254233 father + 0.30348214 mother

Parameter estimates:

| Parameter ϕ | Estimate ϕ | Std. Err. ø | Alternative \$ | DF ¢ | T-Stat ¢ | P-value ϕ |
|------------------|-----------------|-------------|----------------|------|-----------|----------------|
| Intercept | 18.833583 | 3.6094745 | ≠ 0 | 450 | 5.2178184 | <0.0001 |
| father | 0.37254233 | 0.035912028 | ≠ 0 | 450 | 10.373748 | < 0.0001 |
| mother | 0.30348214 | 0.042080619 | ≠ 0 | 450 | 7.2119219 | < 0.0001 |

Analysis of variance table for multiple regression model:

| Source | DF | SS | MS | F-stat | P-value |
|--------|-----|-----------|-----------|-----------|---------|
| Model | 2 | 672.557 | 336.2785 | 82.437552 | <0.0001 |
| Error | 450 | 1835.6358 | 4.0791908 | | |
| Total | 452 | 2508.1928 | | | |

Summary of fit: Root MSE: 2.0197007 R-squared: 0.2681 R-squared (adjusted): 0.2649

For males $R^2 = 23.8\%$, and for females $R^2 = 26.8\%$.

With splitting: for males $R^2 = 23.8\%$, and for females $R^2 = 26.8\%$. Without splitting: for males $R^2 = 23.3\%$, and for females $R^2 = 26.3\%$.

So, we didn't gain much with all that effort. However, it was just clicking an option on the computer, so it really wasn't much effort anyhow.

Go have fun!

MEMORY QUESTIONs









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| STAT 202 Mem | ory Questions | | | | | |
| Combined Sets v | | | | | | |
| To sign the log and 7 errors. You need t | earn credit, you need to work the combined set. You are allowed a o get 50 right in 13 minutes. | maxi | imun | n of | | |
| Click all correct ans | wers, then click submit: | | | | | |
| | No. | | | | | |
| | Yes. | | | | | |
| We expect a correlation. | | | | | | |
| | We don't expect a correlation. | | | | | |
| | SUBMIT | | | | | |
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